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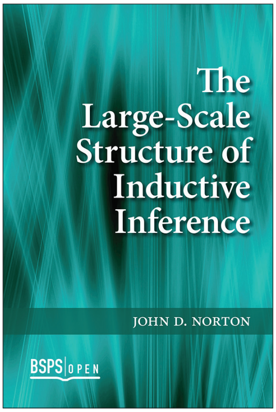

THE LARGE-SCALE STRUCTURE OF INDUCTIVE INFERENCE

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The Recession of the Nebulae

1. Introduction

In 1929, the astronomer Edwin Hubble announced what would become the single most important observation of modern cosmology.[1] Hubble reported that the extragalactic nebulae[2] are receding from us with a velocity proportional to their distance, a result that soon came to be known as "Hubble's law."[3] The establishment of this linear relation seems to be one of the simplest of generalizations. Hubble needed only to compare the velocities of recession and distances to a selection of nebulae, note their linear relation, and declare the result. This is how his affirmation of the linear relationship is often reported in summary. McKenzie's *Major Achievements of Science* describes it thus:

> In 1929 Hubble compared Slipher's determinations of the recession of the nebulae with his own determinations of

1 I thank Siska De Baerdemaeker for helpful comments on an earlier draft.

2 Hubble's "extragalactic nebula" or just "nebula" are the older terms for "galaxy." In 1929, the term "galaxy" referred unambiguously only to our star system, the Milky Way. The Latin *nebula* (plural *nebulae*) means "cloud" and was used by astronomers of Hubble's time to denote the luminous clouds visible in astronomical telescopes. As Hubble explained (1936, 16–17), some of these clouds proved to be gas and dust within our Milky Way. These he called "galactic nebulae." Others were more distant star systems in their own right — "extragalactic nebulae" — that he would just call "nebulae." Hubble defended his reluctance to label these other nebulae "galaxies": "The term *nebula* offers the values of tradition; the term *galaxies*, the glamour . . . of romance" (18; Hubble's emphasis).

3 In 2018, the members of the International Astronomical Union voted to rename the law the "Hubble-Lemaître law."

> distances and he discovered a simple relation now called Hubble's law, that the velocity is proportional to the distance. (1960, 333)

This simple determination seems to be a good illustration of a natural hierarchical structure for inductive support. In it, inductive inferences may proceed only from a lower, more particular level to a higher, more general level.

> *Inductive Hierarchy*
> *Lower level*: velocity and distance assignments to particular nebulae.
> *Higher level*: general relation connecting the velocities and distances of all nebulae.

Hubble's inference, it seems, merely proceeds up the hierarchy. The particulars of a few individual nebulae at the lower level provide inductive support for the general law at the higher level.

Simple as this inference might seem, Hubble's celebrated paper of 1929 showed no respect for this inductive hierarchy. Rather, a multiplicity of inductive inferences moved up and down the hierarchy in an intricate arrangement of interlocking parts, much like those of a complicated geometric puzzle.

To begin, in 1929, Hubble had access to measurements of the velocities of recession of forty-six extragalactic nebulae, but he had independent distance estimates for only twenty-four of them. For these twenty-four, in what initially appears as a simple generalization, he found a linear velocity-distance relation within statistical uncertainties. However, the inference was not a simple generalization since the determination of most of the distances among these twenty-four nebulae depended on assuming hypotheses still needing further support. They are the hypotheses of *Brightest Star Magnitude* and *Clustering of Nebular Luminosity* detailed in Section 3. These hypotheses cannot be located uniquely in the inductive hierarchy above. In the inferences, they are presumed by the determinations of distance, so they are prior to the lower level: that is, still lower. However, the hypotheses accrue support once the inferences of the paper of 1929 are complete. That means that they come at the end of the inferential chain, so they should be placed higher in the inductive hierarchy.

The remaining twenty-two nebulae were more problematic. For them, Hubble had measurements of velocities and apparent luminosities but not distances. He was determined somehow to make use of these data. In doing so, he introduced relations of support that further cut across the inductive hierarchy. This happened in two related ways.

First, Hubble averaged the apparent luminosities of the twenty-two nebulae and computed the average distance associated with them, assuming the *Clustering of Nebular Luminosity* hypothesis and a mean absolute luminosity found in his second, inverted inference (described below). The mean velocity and the mean distance fell within the expectations of the linear relation that he had found for the first twenty-four nebulae, providing further support for that relationship.

Second, Hubble inverted the direction of evidential support. He used the velocity-distance relation itself, in conjunction with the velocities of recession of these twenty-two nebulae, to infer their distances. This inference proceeds down the inductive hierarchy from the higher level to the lower level. He then used the distances computed to determine the absolute luminosities of the twenty-two nebulae. The results provided direct support for the *Clustering of Nebular Luminosity* hypothesis, already used in the earlier analyses.

The overall outcome was a tangle of inductive inferences that failed to respect any simple linear, inductive hierarchy, such as the one indicated above. We shall see that Hubble remarked repeatedly on the agreement among and later the consistency of the results of the inferences as providing the strongest support for his general conclusions. His notion of consistency was much stronger than mere logical compatibility. Rather, it reflects the mutual agreement among the many entangled relations of support. What might be evidence that supports a result in one relation becomes the result supported by evidence in another relation. This agreement among relations of mutual support gives the structure its inductive solidity.

Hubble's analysis also illustrates the use of hypotheses in initiating inductive investigations. The two hypotheses above were used provisionally as warrants since they themselves were not yet fully supported evidentially. Part of Hubble's overall project became the successful discharging of this inductive debt by providing support for these hypotheses.

In Section 2, I will describe how Hubble came to be concerned with the velocities of the nebulae. In Section 3, I will outline the hypotheses that he used in his determinations of the distances to the nebulae. In Sections 4 and 5, I will review the inference to the linear velocity-distance relation for the first twenty-four nebulae. In Section 6, I will review the inverted inferences for the remaining twenty-two nebulae. In Section 7, I will reflect briefly on the strength of support that Hubble could display in 1929 for the linear relationship. In the concluding Section 8, I will summarize the interwoven relations of support in Hubble's paper of 1929. An appendix to this chapter includes technical details of the computations relating absolute and apparent nebular luminosities, known tersely as "magnitudes."

2. Background to Hubble's Investigations

It is now a commonplace of astronomy that space is filled with many immense star systems akin to our own Milky Way. They are the galaxies, as they are now called, or the extragalactic nebulae, as Hubble called them. Yet whether the stars were so distributed in space remained unsettled in the early 1920s. A landmark in the decision was a debate held between the astronomers Harlow Shapley and Heber Curtis on April 26, 1920, at the Smithsonian Museum of Natural History. Shapley defended the view that our Milky Way is the unique great star system of the universe. Curtis, however, argued that our Milky Way is just one of many such "island universes,"[4] as they were then called. The matter was settled fairly quickly. According to Trimble (1995, 1142), it was Hubble himself who provided a cleaner resolution. Starting with observations in 1923,[5] he was able to discern Cepheid variable stars in two nearby nebulae, most notably Andromeda. As we shall see below, this enabled a determination of the distances to these nebulae. They were located outside our Milky Way, he found.

Our solar system has a motion within the Milky Way. With the recognition that our Milky Way is just one of many nebulae, a prosaic question arises: what is the motion of our solar system with respect to these other nebulae? In his later work, *The Realm of the Nebulae*, Hubble (1936, 106–18) recalled how the answer to this question developed. The velocities of nebulae relative to the Earth were known from red shift measurements in the 1910s. The motion of the solar system was then estimated as around 420 mi/sec. The expectation was that, once this motion was subtracted from the motions of the nebulae,

4 The cases each made are published in Shapley and Curtis (1921). See Trimble (1995) for further details.

5 As reported in Hubble (1929b). The results also appeared in a *New York Times* article on December 23, 1924 (Anonymous 1924), and were communicated orally by H.N. Russell at the December–January 1924–25 meeting of the American Astronomical Society (Anonymous 1925).

those motions would be small and random. In particular, there would be as many velocities of approach as of recession. Using a statistical analysis to average away these random motions, we should recover the motion of our solar system with respect to the mean rest state of the nebulae in our vicinity.

As early as 1918, it was already clear that the statistical project was not proceeding smoothly. Wirtz (1918) found the need to add a "*k* term" that corresponded to an overall recession of the nebulae. It meant that the motions of the nebulae visible to us were not distributed randomly about some nebular state of rest. In place of the state of rest was some sort of expansion. The *k* term represented a constant motion of recession from our solar system of 656 km/sec. The motions of the individual nebulae were distributed randomly around that constant motion of recession. Wirtz wrote that,

> If we give this value a verbal interpretation, it is that the system of spiral nebulae disperses [*auseinandertreibt*] with a speed of 656 km [per second] in relation to the momentary position of the solar system as a center. (115)

Over the next decade, Wirtz and others refined the correction term, allowing it to be a function of distance from our solar system. Hubble's celebrated paper of 1929 was a direct contribution to this literature. Its first paragraph identifies the issue to be addressed:

> Determinations of the motion of the sun with respect to the extra-galactic nebulae have involved a K term of several hundred kilometers [per second] which appears to be variable. Explanations of this paradox have been sought in a correlation between apparent radial velocities and distances, but so far the results have not been convincing. The present paper is a re-examination of the question, based on only those nebular distances which are believed to be fairly reliable. (1929a, 168)

The result announced (170–71) was that a statistical fit gave the overall motion of the nebulae as distributed, with some considerable deviations, around a velocity of recession that increases linearly with distance from us. In more detail, the best estimate of the motion of our solar system is 280 km/sec, and, when this is subtracted from the motions of the nebulae, their motions are scattered around an average recessional velocity of 500 km/sec for each million parsec (Mpc) of distance.[6]

A prosaic question about the motion of our solar system had led Hubble to the single most important observational result of modern cosmology.

3. The Determination of Distances

To carry out the analysis of his paper of 1929, Hubble needed determinations of both velocities of and distances to the nebulae. For the forty-six nebulae of his analysis, the velocity determinations proved to be relatively unproblematic. They were determinable from frequency shifts in the spectra of light from the nebulae. The shifts were immediately interpreted as the results of radial velocities: that is, motions along the lines of sight to each nebula.[7] As Hubble (1936, 102–05) recounts, Vesto Slipher, working at the Lowell Observatory, had begun the arduous work of measuring these shifts in 1912. By 1925, he had provided the velocities of twenty-five nebulae.

The locus of difficulty in the analysis was the determination of distances. Two means were available for determining these distances. One was the angular size of the nebula. Nearby nebulae are large: Andromeda extends over 3° in the sky, six times the extent of the full Moon. If we know the absolute size of the nebula in, say, light years, then the distance to the nebula is immediately determined by elementary geometry.

This means of determining distance to the nebulae is not mentioned in Hubble's paper (1929a).[8] Rather, Hubble explicitly reports only luminosity-based determinations. They depend on the fact that the intensity of light emitted by a celestial object diminishes with the inverse square of distance.

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6 This value of 500 km/sec.Mpc of what we now call the Hubble constant proved to be about an order of magnitude too large as a result of systematic errors in Hubble's determinations of distances. By 1958, the value had been reduced by Sandage to a more modern value of 75 km/sec. Mpc, which corresponded to a Hubble age of the universe of $1.3x10^9$ years.

7 Slipher (1912, 56) wrote that, "the velocity-like displacement might not be due to some other cause, but I believe we have at the present no other interpretation for it. Hence we may conclude that the Andromeda Nebula is approaching the solar system with a velocity of about 300 kilometers per second." Hubble (1936, 34) held the same view but more cautiously: "Although no other plausible explanation of redshifts has been found, the interpretation as velocity-shifts may be considered as a theory still to be tested by actual observations."

8 Hubble and Humason (1931, 52) recount that the difficulty with the method is that the brightnesses of the nebulae fade as we move away from their centers, so that different photographic exposures of the same nebula give different sizes.

Thus, if we know the absolute magnitude of the object's luminosity, then we can determine its distance: we compare this absolute magnitude with the apparent magnitude that we perceive, either visually or photographically.

The weakness of this approach is that the absolute magnitudes are hard to determine; direct measurements give us only apparent magnitudes. Without some independent means of determining the absolute magnitude, the approach cannot be applied. In his paper of 1929, Hubble relied on three methods of determining absolute magnitude.

1. *Cepheid Variable Stars*. Henrietta Leavitt (1912) had reported that certain stars in the Magellenic Clouds varied periodically in magnitude and that there was a definite relationship between the period and the magnitude. Subsequent parallax measurements to other Cepheid variable stars enabled determinations of their distances and thus their absolute magnitudes. Combining these results meant that an observation of the period of one of these variable stars enabled a determination of its absolute magnitude and thus its distance. Hubble himself used this method in 1923 in his determination of the distance to the nebula Andromeda. The distinctive shape of the curve[9] plotting the change of visual magnitude with time enabled him to identify the variable stars that he found in Andromeda as Cepheid variable stars. This was, Hubble (1936, 16) reported, the first reliable method of determining distances to nebulae. It was also the most reliable of the three methods of the paper of 1929 but could be applied only if a Cepheid variable star could be resolved in the nebula.

2. *Brightest Star Magnitude*. It seemed reasonable to assume that different nebulae are constituted of the same sorts of stars, with the same range of possible magnitudes. That led to the expectation that the brightest stars in each nebula have the same absolute magnitude.[10] Hubble (1929a, 168) offered

9 Shown in Hubble (1936, 95).

10 Hubble footnoted an earlier paper (1926) in which he had already advanced the hypothesis (357–61), although only hesitantly, as a "reasonable assumption, supported by such evidence as is available" (357).

an absolute magnitude determined photographically of $M = -6.3$ (see the appendix for a review of the system of units used for apparent and absolute magnitudes). This assumption is important in untangling the evidential relations displayed in Hubble's paper. So I will display it as a hypothesis to which we will return:

Brightest Star Magnitude. The brightest stars in each nebula have the same absolute magnitude.

Hubble approached the hypothesis with optimism and caution:

> The apparent luminosities of the brightest stars in such nebulae are thus criteria which, although rough and to be applied with caution, furnish reasonable estimates of the distances of all extra-galactic systems in which even a few stars can be detected. (1929a, 168–69)

Hubble conceded the limitation that the method could be applied only to nebulae close enough for individual stars to be resolved telescopically. The third method was untroubled by this limitation.

3. *Clustering of Nebular Luminosity.* Drawing from his earlier survey of nebulae (Hubble 1926), he suggested that the absolute magnitudes of nebulae were similar insofar as they were distributed randomly but not too distant from their average. The average value offered (1929a, 169) is a visually determined magnitude of $M = -15.2$ (recall from the appendix that smaller magnitudes correspond to greater brightnesses. A magnitude of –15 is very bright). Actual values, Hubble reported, are "exhibiting a range of four or five magnitudes about [this] average" (169). Once again, this assumption will play an important role in the evidential relations and is displayed thus:

> *Clustering of Nebular Luminosity*. The absolute magnitudes of nebulae cluster in a small interval of four or five units of magnitude about a single mean common to all nebulae.

Four to five units of magnitude amount to a considerable error if we are trying to estimate the distance to just one nebula. It is shown in the appendix that this uncertainty in the absolute magnitude of any particular nebula introduces an uncertainty in the determination of distance of roughly one order of magnitude: that is, the extremes of the full range differ by a factor of 10.

These deviations can be averaged away if we aggregate data from many nebulae so that we can recover more reliable distance determinations for averages. This is especially helpful in getting a more accurate distance estimate to a cluster of nebulae whose members are assumed to be grouped around the same location in space. Hubble (1929a, 169) explains that he would use this averaging technique:

> The application of this statistical average [M = –15.2] to individual cases can rarely be used to advantage, but where considerable numbers are involved, and especially in the various clusters of nebulae, mean apparent luminosities of the nebulae themselves offer reliable estimates of the mean distances.

Hubble (1929a) says little more on the use of this technique. Hubble and Humason (1931) is a lengthier and more detailed exposition, using considerably more data. There we find how effective the averaging can be. They report clusters consisting almost always of several hundred nebulae, up to a maximum of 800.[11]

To determine the distance to some particular nebula in a cluster, they would survey the full range of apparent magnitudes of the nebulae in the cluster. The aggregation of survey data greatly reduces errors. For example, consider a cluster of 400 nebulae whose magnitudes are spread over an interval of 4 or 5 magnitudes around the true mean of –15.2. The spread of the average of the magnitudes of the cluster around that true mean is reduced by

11 A table in Hubble and Humason (1931, 74) lists the numbers of nebulae in named clusters as Virgo-(500), Pegasus-100, Pisces-20, Cancer-150, Perseus-500, Coma-800, Ursa Major-300, and Leo-400. Whatever hesitation is flagged by the parentheses for the Virgo cluster, Hubble (1936, 54) reports "several hundred" nebulae in it.

a factor of 400 = 20. We find in the appendix that this reduces the interval to 0.25 magnitudes and corresponds to an error in distance estimates in which the farthest distance is merely 12% greater than the nearest distance. This provides a good determination of the absolute magnitude of and distance to a nebula whose brightness matches the average.[12] That distance is then also the estimate of the distance to the particular nebula of interest.

4. From Particulars to Generalities

Although forty-six nebulae were included in Hubble's (1929a) analysis, Hubble was able to estimate individual distances to only twenty-four of them. He inferred the linear relation between their distances and velocities by directly comparing distances and velocities. He reported the results of two ways of arriving at the linear relation.

The first, and most direct, way took the velocities and distances of the individual nebulae and used standard statistical methods to find the best fit of a relation written in more modern vector notation as

$$\mathbf{v}_i = \mathbf{r}_i K + \mathbf{V}_0$$

Here $\mathbf{v}_i$ is the vector velocity of the ith nebula located a vector displacement $\mathbf{r}_i$ from us and $\mathbf{V}_0$ is the vector velocity of our solar system. The constant K is now known as the "Hubble constant" and is the parameter of greatest interest to us now. It converts a scalar distance r to a nebula to its scalar velocity of recession $v = Kr$. The velocity $\mathbf{v}_i$ is not the velocity observed from the Earth through the red shift, for those observations are taken from a vantage point itself moving at $\mathbf{V}_0$. The velocity that we observe for the ith nebula is the difference $\mathbf{v}_i - \mathbf{V}_0$. Hubble reported that the best fit gave

$$K = 465 \pm 50 \text{ km/sec.Mpc} \quad V_0 = 306 \text{ km/sec} \quad A = 286^\circ \quad D = 40^\circ$$

The second way proceeded by first reducing the data for the twenty-four nebulae to nine groupings and first averaging within each grouping. Hubble indicated only that the groupings were selected "according to proximity in direction and in distance" (1929a, 170). Presumably, the effect of the

12 Hubble and Humason (1931, 56) summarize the strategy as "the mean or most frequent apparent magnitude of the many members [of a cluster] is a good indication of the distance of a cluster, and hence clusters offer the greatest distances that can definitely be assigned to individual objects."

averaging, once again, was to reduce the effect of random deviations from linearity, this time prior to finding the statistical best fit of the above relation. The index i would now refer to the ith group. Hubble reported that best fit as

$$K = 513 \pm 60 \text{ km/sec.Mpc} \quad V_0 = 247 \text{ km/sec} \quad A = 269^{\circ} \quad D = 33^{\circ}$$

For his final result, Hubble selected values intermediate between these two sets and rounded them:[13]

$$K = 500 \text{ km/sec.Mpc} \quad V_0 = 280 \text{ km/sec} \quad A = 277^{\circ} \quad D = 36^{\circ}$$

Since the solar velocity $\mathbf{V}_0$ is comparable in size to the nebular velocities $\mathbf{v}_i$, Hubble's analysis had to pass through the more indirect route of finding the best fit of the above relation. Merely computing the ratio of observed velocity and distance for each nebula would have omitted the essential correction for the Earth's motion. Hubble's figure, redrawn here as Figure 7.1, gives a sense of the large size of the residuals that deviate from his best-fit relations. It displays the velocities of nebulae, after the velocity of our solar system has been subtracted, in relation to their distances.

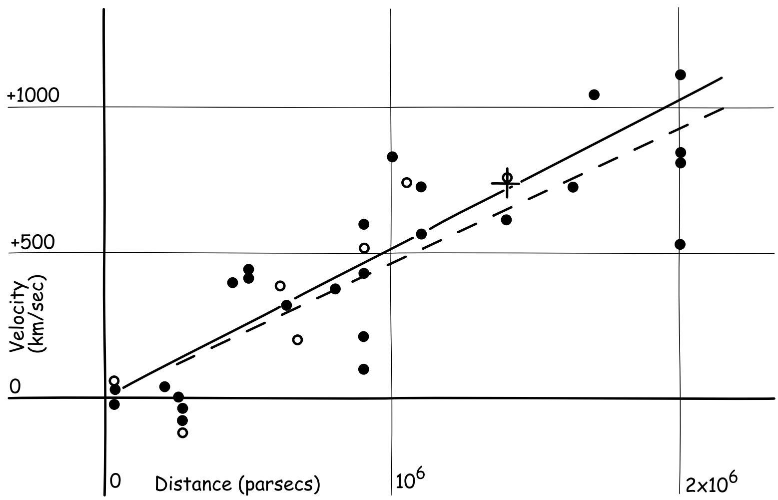

Figure 7.1. Hubble's "Velocity-Distance Relation among Extra-Galactic Nebulae"

13 Hubble converted the celestial coordinates into galactic coordinates: longitude 32°, latitude +18°.

An extended caption explains the data presented. Hubble (1929a, 172) writes that

> The black discs and full line represent the solution for solar motion using the nebulae individually; the circles and broken line represent the solution combining the nebulae into groups. . . .

There are twenty-four black discs, and they correspond loosely[14] to the data in Table 7.1 for twenty-four nebulae whose distances can be determined. Hubble concluded that

> . . . the cross represents the mean velocity corresponding to the mean distance of 22 nebulae whose distances could not be estimated individually.

I will return to the treatment of these twenty-two nebulae in Section 6.

5. Hubble's Hypotheses

The appearance of this last inference is of a traditional generalization that proceeds from the particulars of the lower level to the covering generality at the higher level of the hierarchy indicated in Section 1. The appearance is deceptive, for most of the distance determinations in the particulars depend on the hypotheses indicated in Section 3. Since the subsequent generalizations depended on them, the generalization was not secure until Hubble provided further evidence in support of the hypotheses. This stage of his investigation took on an inductive debt. We shall see that Hubble continued the analysis in a way intended to discharge some of that debt.

The data for these twenty-four nebulae were presented in Table 1 of Hubble's paper (1929a), reproduced here as Table 7.1.

To arrive at the distances in this table, Hubble used all three of the methods discussed above. He did not lay out the specifics of the determinations in each case. All of the details would be lengthy and not fit into the short announcement that he offered. Hubble and Humason (1931) provide a similar

14 We should not expect the velocities in the figure to match those of Table 7.1 up to a constant subtractive factor. The correction for solar motion is a vector subtraction whose scalar effect will vary according to the differences in the directions of the vectors in the subtraction.

Table 7.1. Hubble's "Nebulae Whose Distances Have Been Estimated from Stars Involved or from Mean Luminosities in a Cluster"

	Object	m_s photographic magnitude of brightest stars	r distance* in megaparsecs	v velocity km/sec	m_t visual magnitude	M_t absolute visual magnitude computed† from r, m_t
1	Small Magellenic	..	0.032	+170	1.5	–16.0
2	Large Magellenic	..	0.034	+290	0.5	–17.2
3	NGC 6822	..	0.214	–130	9.0	–12.7
4	NGC 598	..	0.263	–70	7.0	–15.1
5	NGC 221	..	0.275	–185	8.8	–13.4
6	NGC 224	..	0.275	–220	5.0	–17.2
7	NGC 5457	17.0	0.45	+200	9.9	–13.3
8	NGC 4736	17.3	0.5	+290	8.4	–15.1
9	NGC 5194	17.3	0.5	+270	7.4	–16.1
10	NGC 4449	17.8	0.63	+200	9.5	–14.5
11	NGC 4214	18.3	0.8	+300	11.3	–13.2
12	NGC 3031	18.5	0.9	–30	8.3	–16.4
13	NGC 3627	18.5	0.9	+650	9.1	–15.7
14	NGC 4826	18.5	0.9	+150	9.0	–15.7
15	NGC 5236	18.5	0.9	+500	10.4	–14.4
16	NGC 1068	18.7	1.0	+920	9.1	–15.9
17	NGC 5055	19.0	1.1	+450	9.6	–15.6
18	NGC 7331	19.0	1.1	+500	10.4	–14.8
19	NGC 4258	19.5	1.4	+500	8.7	–17.0
20	NGC 4151	20.0	1.7	+960	12.0	–14.2
21	NGC 4382	..	2.0	+500	10.0	–16.5
22	NGC 4472	..	2.0	+850	8.8	–17.7
23	NGC 4486	..	2.0	+800	9.7	–16.8
24	NGC 4649	..	2.0	+1,090	9.5	–17.0
	NGC = nebula number in the New General Catalog					mean –15.5

* These distances are systematically low. Hubble reports 0.275 Mpc for the distance to nearby Andromeda, whereas the more recent estimate is 0.780 Mpc.

† Using formula (A3) of the appendix. The table has distances in units of megaparsecs, whereas distance in (A3) are entered in parsecs.

analysis, with more data and details, which has to be considerably lengthier and more complicated in its reporting. In his report, Hubble (1929a, 170) limited himself to general statements:

> The first seven distances are the most reliable, depending, except for M32 [= NGC 221] the companion of M31 [= Andromeda, NGC 224], upon extensive investigations of many stars involved.

For Andromeda (M31 = NGC 224), we know from Hubble (1929b) that he used Cepheid variable stars for the distance determination. Presumably, the *Brightest Star Magnitude* hypothesis was not used in the distance estimates for these first seven objects since there are no brightest star magnitude entries for them. Subsequent distance estimates did consider the magnitudes of the brightest stars since they are given for rows seven to twenty. Hubble continued:

> The next thirteen distances,[15] depending upon the criterion of a uniform upper limit of stellar luminosity, are subject to considerable probable errors but are believed to be the most reasonable values at present available. (1929a, 170)

The use of mean nebular magnitudes for distance determination is finally mentioned for rows twenty-one to twenty-four:

> The last four objects appear to be in the Virgo Cluster. The distance assigned to the cluster, 2 x 10^6 parsecs, is derived from the distribution of nebular luminosities, together with luminosities of stars in some of the later-type spirals, and differs somewhat from the Harvard estimate of ten million light years. (170)

Here the *Clustering of Nebular Luminosity* hypothesis was employed. That it had a larger role is suggested by the title given to the table as a whole: "Distances . . . from Mean Luminosities in a Cluster."

15 Presumably, he means the "next fourteen," rows seven to twenty.

6. From Generalities to Particulars

Hubble then turned to the remaining twenty-two nebulae for which velocities were known but not distances. He was intent on recovering some evidential import from the data. The data with which he worked are presented in Table 7.2, which reproduces his Table 2 (1929a). The column v is the velocity determined by red shifts for the nebula with the indicated NGC number. The next column v_s indicates the correction that must be subtracted from the observed velocity to correct for solar motion.

With these data in hand, Hubble proceeded with two approaches. The first was the crudest. It simply worked out the velocity-distance relation for the average behavior of all of the twenty-two nebulae. Since the velocity-distance relation is presumed to be linear, it should hold for the average of the velocities and distances. Hubble found an average velocity of 745 km/sec and an average distance of 1.4 Mpc. These averaged data then give an estimate for the constant $K = 745/1.4 \approx 530$ km/sec.Mpc. Given the magnitude of errors likely (see below), the agreement was likely well within error limits for the value of 500 km/sec.Mpc estimated in the earlier part of the paper.

For my purposes, it is interesting to see that even here Hubble's analysis relied on the *Clustering of Nebular Luminosity* hypothesis. It was not needed to recover the average velocity. That was simple arithmetic.[16] The hypothesis was needed to determine the average distance. According to the hypothesis, the absolute magnitudes of the individual nebulae varied at an interval of 4 to 5 magnitudes around the common mean value. This range would then be reflected in the apparent magnitudes reported in the column m_t of Table 7.2. However, taking the average of the apparent magnitudes reduces the interval by a factor of $1/\sqrt{22} = 1/4.69$ to an interval of roughly the size of a single magnitude. We find in the appendix that the farthest distance in the associated distance interval is 58% greater than the nearest distance. The average apparent magnitude of 10.5 is far from the absolute magnitude of –15.3 assumed.[17] The diminution is entirely the result of the great distance associated with the average. That distance is computed[18] from (A3) and is 1.445 Mpc.

16 (Average $v = 748.4$) – (average correction $v_s = 2.95$) = 745.4 km/sec.

17 This absolute magnitude of –15.3 is recovered from the next stage of calculations on these twenty-two nebulae.

18 That is $\log_{10} d = 0.2(10.5 + 15.3) + 1 = 6.16$, so that $d = 10^{6.16} = 1.445 \times 10^6$ pc.

The more elaborate of the two approaches involved using the velocity-distance relation in reverse. Starting with the corrected velocity, $v - v_s$, for each of the twenty-two nebulae, Hubble computed the distance r that the linear velocity-distance relation required, where he assumed a value for the K constant of 500 km/sec.Mpc. The results are reported in the r column of Table 7.2 and conform to the formula $r = (v - v_s)/500$. Since these distances were computed using the very relation under scrutiny, by themselves they could provide no evidence for the relation. To extract some useful evidential import, Hubble used these distances r to calculate[19] the absolute magnitude M_t of each nebula from the measured, apparent magnitude, m_t. The results are reported in the last column of Table 7.2. Hubble computed the mean to be –15.3.

What he found notable was that the mean absolute magnitude computed for these twenty-two nebulae matched almost exactly the mean –15.5 computed for the first twenty-four nebulae using their independently known distances. Similarly, their ranges agreed: 4.9 for the twenty-two nebulae of Table 7.2[20] and 5.0 for the twenty-four nebulae of Table 7.1. The most direct reading is that the new results from the twenty-two nebulae provide another instance of the *Clustering of Nebular Luminosity* hypothesis, using the same mean and range as the earlier analysis. This provides direct support for the hypothesis. Hubble was more celebratory and expansive in his assessment:

> The two mean magnitudes, –15.3 and –15.5, the ranges, 4.9 and 5.0 mag., and the frequency distributions are closely similar for these two entirely independent sets of data; and even the slight difference in mean magnitudes can be attributed to the selected, very bright, nebulae in the Virgo Cluster. This entirely unforced agreement supports the validity of the velocity-distance relation in a very evident matter. Finally, it is worth recording that the frequency distribution of absolute magnitudes in the two tables combined is comparable with those found in the various clusters of nebulae. (1929a, 172–73)

19 The calculation employed formula (A3) of the appendix. Note that d in that formula is in parsecs, whereas r in Table 7.2 is in megaparsecs.

20 I find the range to be 4.8, extending from –12.8 for NGC 1700 to –17.6 for NGC 4594.

Table 7.2. Hubble's "Nebulae Whose Distances Are Estimated from Radial Velocities"

	NGC nebula number	v Velocity km/sec	v_s Velocity correction subtracted for solar motion	r Distance Mpc	m_t Apparent magnitude	M_t Absolute magnitude computed from r, m_t
	278	650	–110	1.52	12.0	–13.9
	404	–25	–65	..	11.1	..
	584	1,800	75	3.45	10.9	–16.8
	936	1,300	115	2.37	11.1	–15.7
	1023	300	–10	0.62	10.2	–13.8
	1700	800	220	1.16	12.5	–12.8
	2681	700	–10	1.42	10.7	–15.0
	2683	400	65	0.67	9.9	–14.3
	2841	600	–20	1.24	9.4	–16.1
	3034	290	–105	0.79	9	–15.5
	3115	600	105	1	9.5	–15.5
	3368	940	70	1.74	10	–16.2
	3379	810	65	1.49	9.4	–16.4
	3489	600	50	1.1	11.2	–14.0
	3521	730	95	1.27	10.1	–15.4
	3623	800	35	1.53	9.9	–16.0
	4111	800	–95	1.79	10.1	–16.1
	4526	580	–20	1.2	11.1	–14.3
	4565	1,100	–75	2.35	11	–15.9
	4594	1,140	25	2.23	9.1	–17.6
	5005	900	–130	2.06	11.1	–15.5
	5866	650	–215	1.73	11.7	–14.5
Mean		748.4	2.95		10.5	–15.3

7. How Strong Was the Evidence for Linearity?

My concern here is the tangled structure of the relations of inductive support. Although it is independent of this concern, it is worth noting that Hubble's evidence in 1929 for the linear relation was weak. This was so even though his paper of 1929 is routinely celebrated as the origin of the linear relation between the velocities of recession of the nebulae and their distances. A glance at Figure 7.1 shows just how weak was the establishment of the linearity. The data points are so broadly scattered about the straight lines fitted that all that can be inferred securely is that the velocities are increasing with the distances. The difficulty is that nebulae close to our Milky Way have particular motions in random directions of the order of the overall velocity of recession. These motions confound the linear motion of recession. To reveal the linear relation more clearly requires examination of more distant nebulae for which the particular motions become successively smaller in relation to the velocity of recession.

As long as Hubble's interest lay in the original project of determining the motion of our solar system, the weakness of the evidence for linearity is a smaller concern. We might reasonably expect that other velocity-distance relations compatible with the data would have only a minor effect on the estimates of solar motion. The threat is more serious, however, if Hubble's paper is to underwrite the founding empirical observation of modern cosmology: the linearity of the velocity-distance relation.

Hubble had a response to this threat in his paper. He allowed that his data merely "establish a roughly linear relation" (1929a, 173). The solution lay in an extension to more distant nebulae and was already under way. Hubble reported a result for NGC 7619, whose distance he estimated at roughly 7 Mpc. That greatly exceeded the distance of 1 or 2 Mpc of nebulae investigated so far. Its speed of recession still fit well enough with his *K* factor of 500. Shortly after, in joint work, Hubble and Humason (1931) reported on velocities of recession of still more distant nebulae. Their Figure 5 (77) plotted data for nebular clusters, one of which is more than 30 Mpc distant. In this plot, the linearity of the paper of 1929 survives. Hubble and Humason had become so confident of the linear relationship that they proposed its use to determine distances. It was, they boasted,

> . . . a new method of determining distances of individual objects in which the percentage errors actually diminish with distance. (76)

This remark foreshadowed the recent practice of identifying the locations of distant galaxies merely by citing their red shift factors directly. Red shift has become the surrogate for distance.

By the time of his more popular work in 1936, Hubble reasserted his confidence that the linearity of the relation had been vindicated. He wrote of the success of the extension of the investigation to more distant nebulae:

> The relation is plausible but not unique. The true relation might be a curve which was nearly linear within the range covered by the observations, but which departed widely from a straight line in the regions beyond the faintest nebulae in the group. This possibility was investigated by extrapolating the adopted relation extending it far out into the hitherto unobserved regions and testing it by new observations. Such a procedure often leads to minor, or even to major, revisions in the relation first selected: it has been said that research proceeds by successive approximations. However, in the investigation of red-shifts, no revision was definitely indicated. The linear relation has survived repeated tests of this nature and is known to hold, at least approximately, as far out into space as the observations can be carried with existing instruments. (3–4)

8. Conclusion and Summary

In the introduction, I sketched the inductive hierarchy to which one might assume that Hubble's inferences of 1929 conformed. We have now seen that his inductive inferences did not respect this hierarchy. Rather, his inferences are interwoven nonhierarchically through the following sets of propositions.

(a) Sets of velocities of recession assigned to nebulae
(b) Sets of distances assigned to nebulae
(c) Linear relations asserted between their velocities and distances

(d) Hypothesis of *Brightest Star Magnitude*
(e) Hypothesis of *Clustering of Nebular Luminosity*

The inferences were as follows:

(i) In Sections 4 and 5, we saw inferences from the sets of velocity (a) and distance (b) assignments to a linear relationship (c), in which many of the distance assignments already presumed the two hypotheses (d) and (e).

(ii) In Section 6, we saw an inference from the means of the velocities (a) and distances (b) to an instance of the linear relationship (c). The determination of the mean distance once again presumed hypothesis (e) as well as a mean absolute magnitude for nebulae determined by the inferences of (iv).

(iii) In Section 6, we saw an inference from sets of velocity assignments (a) and the linear relationship (c) to sets of distance assignments (b).

(iv) In Section 6, Hubble proceeded from the distances computed in (iii) and inferred to a set of absolute magnitudes that affirmed hypothesis (e).

Use of the velocity-distance relation in (iii) to infer back to distances became a fixture in astronomy. In his more popular work, Hubble (1936, 115) was confident enough of this inference that he wrote

> The velocity-distance relation, once established, could evidently be used as a criterion of distance for *all* nebulae whose velocities were known.

This inference appears initially as the mere recovery of a deductive consequence of the velocity-distance relation. It also has an inductive component. I have emphasized the "all" since it includes the nebulae originally used to establish the velocity-distance relation. We gain inductive support for an independently determined distance to some nebula if we find that it conforms to the velocity-distance relation. Alternatively, if conformity fails, then we have a check and a correction for the original distance determination.

The cogency of Hubble's inferences required that strong evidential support be provided for hypotheses (d) and (e), or the distance determinations of his analysis would be compromised. Discharging this inductive debt was an obligation taken seriously in the later analysis of Hubble and Humason (1931). Of its thirty-eight pages, six were devoted to a section on the "Upper Limit of Stellar Luminosity as a Criterion of Distance" (46–51), and another five pages were devoted to a section on the "Total Luminosity of Nebulae as a Criterion of Distance" (52–56). That is, almost 30% of the paper was spent elaborating and establishing these two hypotheses.

More generally, Hubble repeatedly offered the agreement among the results of all of these inferences as giving general support to his analysis. We have already seen his remark that "this entirely unforced agreement supports the validity of the velocity-distance relation in a very evident matter" (1929a, 172–73).[21] Hubble and Humason (1931, 43) commence their paper by defending their methods of determining nebular distance, whose initiating assumption is "supported in a general way by the consistency of the results to which it leads." Later they announce that, "since the two investigations were based upon different criteria of distance, the close agreement emphasizes the internal consistency of our present ideas concerning luminosities of nebulae" (76).

In his more popular narrative, Hubble (1936, 101) reflected on the various criteria used to determine nebular distances, including the velocity-distance relation itself, and he concluded that

> The exploration of the realm of the nebulae was carried out with the aid of these criteria. The early work was justified largely by the internal consistency of the results. The foundations were firmly established, but the superstructure represented considerable extrapolations. These were tested in every way that could be devised, but the tests for the most part concerned internal consistency. The ultimate acceptance of the

21 Hubble (1929a) does not provide further evidence explicitly and specifically supporting the *Brightest Star Magnitude* hypothesis. Perhaps this unforced agreement provides independent support for the nebular distances determined using this hypothesis and thus, indirectly, support for the hypothesis itself.

> superstructure was due to the steady accumulation of consistent results rather than to critical and definitive experiments.

A few pages later Hubble reflected on the use of distances derived from the mean and range of the absolute luminosities in establishing the velocity-distance relation:

> The consistency of these results was additional evidence of the validity of the velocity-distance relation. (115)

The consistency so important to Hubble is not the consistency of deductive logic, in which it merely designates a lack of contradiction. This deductive sense of consistency by itself provides no inductive support. The Hubble law of expansion of the nebulae in our universe is logically consistent with the existence of another, parallel universe, isolated from ours, in which nebulae approach each other. The fact of logical consistency supplies no inductive support for the existence of such a parallel universe.

The consistency alluded to by Hubble was the agreement among the many entangled relations of inductive support of his analysis. The Hubble law itself in one part is inductively supported by other results and in another part is used to provide inductive support. The hypotheses of *Brightest Star Magnitude* and *Clustering of Nebular Luminosity* are used, in one part, to warrant inductive inferences to other results, and in another part the results are supported by inductive inferences. The overall import is that no proposition within Hubble's analysis is left without inductive support, and that fact gives his analysis its inductive solidity.

Appendix: Luminosity and Magnitude

Hubble's accounts above discuss the brightness of stars and nebulae using the standard system of magnitudes. His paper of 1929 was written for experts, so Hubble had no need there to explain the system. His more popular *The Realm of the Nebulae* (1936, 9–13), however, describes the system. The luminosity L of an object is the rate at which it emits luminous energy. Our perception of brightness associates equal increments in brightness to equal multiples of luminosity. Thus, the brightness of an object is given by a logarithmic function of the luminosity. That is, the apparent magnitudes m_1 and m_2 of two objects at the same distance from us are related to their luminosities L_1 and L_2 by

$$m_1 - m_2 = -2.5 \log_{10} (L_1/L_2) \tag{A1}$$

The minus sign in the relation means that a *brighter* object has a *smaller* magnitude.

This particular logarithmic relation was chosen to preserve continuity with the ancient visual system of reporting star brightnesses, already found in Ptolemy's *Almagest*. There stars were grouped by their brightnesses into six magnitudes. The first magnitude was the brightest and the sixth the dimmest visible. If the associated luminosities are L_1, L_2, . . . L_6, then stepping through them represents equal increases in apparent brightness as long as

$$L_1/L_2 = L_2/L_3 = L_3/L_4 = L_4/L_5 = L_5/L_6 = 2.5$$

The ratio of 2.5 arises from the stipulation that the full range of luminosities spans 100 to 1: that is, $L_1/L_6 = 100$. Thus, each of the five steps corresponds to a multiplicative factor of $100^{1/5} = 2.512$, rounded down to 2.5. The magnitudes are labeled "visual" or "photographic" according to the media with which they are measured. The distinction is important since the two media have different sensitivities to different frequencies of light.

The apparent brightness of an object diminishes with the inverse square of distance from us. If the two objects in formula (A1) were removed to distances d_1 and d_2 respectively, then the ratio (L_1/L_2) must be replaced by the ratio (L_1/d_1^2) / (L_2/d_2^2). The relation among apparent magnitudes becomes

$$m_1 - m_2 = -2.5 \log_{10} (L_1/L_2)\, (d_2^2/d_1^2) \tag{A2}$$

The absolute magnitude of an object M is stipulated to be the apparent magnitude that the object would have were it placed 10 parsecs from us.[22] Using only the distance dependency in (A2), it follows that the apparent magnitude m of an object of absolute magnitude M at a distance of d parsecs is[23]

$$m = M + 5 \log_{10} d - 5 \quad \text{or} \quad \log_{10} d = 0.2(m - M) + 1 \tag{A3}$$

Hubble (1929a) supposes that the intrinsic brightnesses of all nebulae are within 4 to 5 absolute magnitudes of each other. Assuming a mean absolute magnitude for some nebula will lead to errors in distance estimates. To take

22 A parsec is the distance at which the mean Earth-Sun distance subtends one second of arc. It is a convenient astronomical unit since distances to nearby stars are revealed by their parallax during the Earth's annual motion around the Sun. 1 parsec = 3.258 light years. A megaparsec or Mpc is 1 million parsecs.

23 Set $d_2 = 10$ and $d_1 = d$; note that $\log_{10} (d^2/10^2) = 2 \log_{10} d - 2 \log_{10} 10 = 2 \log_{10} d - 2$.

the most extreme case, an apparent magnitude m can derive from an object with absolute magnitude M_1 at distance d_1 or another object with absolute magnitude M_2 at distance d_2, where $M_1 - M_2 = 5$. Thus, we have from (A3) that

$$M_1 + 5 \log_{10} d_1 = M_2 + 5 \log_{10} d_2$$

and then

$$5 = M_1 - M_2 = 5 \log_{10} (d_2/d_1)$$

It follows that $\log_{10} (d_2/d_1) = 1$, so that $d_2/d_1 = 10$. That is, the uncertainty in the absolute magnitudes of nebulae corresponds to an uncertainty of one order of magnitude in their spatial distances.

If, however, we follow Hubble's technique of averaging, then this uncertainty is greatly reduced in estimating the value of the true mean.[24] For a cluster of 400 nebulae, the spread of the mean is reduced by a factor of $1/\sqrt{400} = 1/20 = 0.05$. So the spread is $5 \times 0.05 = 0.25$. Thus, we have from (A3) as before

$$0.25 = M_1 - M_2 = 5 \log_{10} (d_2/d_1)$$

We now have for the corresponding distances that $\log_{10} (d_2/d_1) = 0.05$, so that $d_2/d_1 = 1.122$. That is, the farthest distance of the associated interval of distances is merely 12% greater than the nearest distance.

For a group of twenty-two nebulae, the spread of the mean reduces by a factor of $1/\sqrt{22} = 1/4.69$. If we approximate the spread of 4 to 5 magnitudes to be reduced to one order of magnitude, then we have from (A3) that

$$1 = M_1 - M_2 = 5 \log_{10} (d_2/d_1)$$

We now have $\log_{10} (d_2/d_1) = 0.2$, so that $d_2/d_1 = 1.585$. That is, the farthest distance of the associated interval of distances is 58% greater than the nearest distance.

24 Assume that we have $n = 400$ independent samples from the same distribution with variance σ^2. The variance of the mean is σ^2/n. Hence, the standard deviation is $\sigma/\sqrt{n}$.

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