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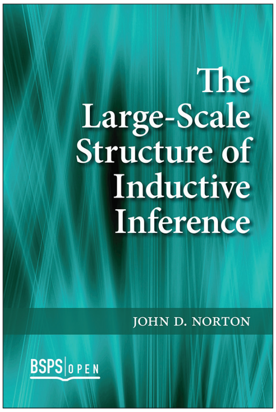

THE LARGE-SCALE STRUCTURE OF INDUCTIVE INFERENCE

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Newton on Universal Gravitation

1. Introduction

Isaac Newton's reasoning in his seventeenth-century *Mathematical Principles of Natural Philosophy* ([1726] 1962) remains to this day a model of tight, carefully controlled argumentation. Its inductive centerpiece lays out the evidential case for his theory of universal gravitation with exemplary caution and discipline. Within his argumentation, there are two cases of pairs of propositions in which relations of inductive support cross over each other, in analogy to the relations of structural support in an arch. The first pair comprises the two core propositions of Newton's celebrated "Moon test." The second pair comprises the propositions of an inverse square law of gravity and of the elliptical orbits of the planets.

In both cases, the individual relations of support have the following structure: the observed evidence supports a proposition by means of a warranting hypothesis. Schematically, this can be written as

Observed evidence

(warrant) Hypothesis

________________(deduce)

Proposition

The crossing over of relations of support arises in both cases in the following way. We have two propositions, proposition_1 and proposition_2, such that

Observed evidence

(warrant) Proposition_1

______________(deduce)

Proposition_2

Observed evidence

(warrant) Proposition_2

______________(deduce)

Proposition_2

Finally, each of the individual inferences above is deductive. They combine to give a totality in which the observed evidence inductively supports both propositions. That is, the relations of support are locally deductive but inductive in their combination.

Observed evidence

______________(induction)

Proposition_1 & Proposition_2

The two examples are treated in turn in the sections that follow.

2. The Moon Test

One of Newton's more remarkable discoveries in his theory of universal gravitation is the identity of two forces. The first is the celestial force that deflects planets into orbit around the Sun and deflects moons into orbits around their planets. The second is the force of gravity that leads to the fall of free bodies at the Earth's surface, such as hurled stones. That these forces are the same is now a commonplace. It was a major discovery in the seventeenth century, for the ancient tradition had been that the physics of terrestrial bodies differs from the physics of celestial matter. Newton needed a strong argument to establish the identity.

The identity of the two forces was established early by Newton in Book III of his *Principia* ([1726] 1962). That book presents a sequence of propositions laying out his argument for universal gravitation. The first three propositions establish that the celestial force of attraction acting on an orbiting body varies with the inverse square of distance from the center of the attracting body in three cases: the orbits of Jupiter's moons about the center of Jupiter, the orbits of the planets about the Sun's center, and the orbit of the Moon about the Earth's center. The fourth proposition asserts the identity of terrestrial gravity and the celestial force acting on the Earth's Moon.

To arrive at this fourth proposition, Newton determined the acceleration of the Moon toward the Earth. It is this acceleration that deflects the Moon from its linear, inertial motion and brings it into orbit around the Earth. We would now represent this acceleration directly as so many feet/second^2 or meters/second^2. Newton proceeded indirectly. A body falling with constant acceleration a from rest will cover a distance $at^2/2$ in time t. Newton used this distance as the measure of acceleration.

As a result of its orbital motion, Newton noted, the Moon falls 15 Paris feet 1 inch 1 4/9 lines (1/12 of an inch) in one minute. That is, it falls 15.0934 Paris feet in one minute. The Moon is roughly 60 times farther away from the center of the Earth than a point on the Earth's surface. Hence, if the celestial force acting on the Moon is governed by an inverse square law all the way down to the Earth's surface, then it would be 60^2 times greater on the Earth's surface. This means that a body falling under its action at the Earth's surface would fall 15.0934 x 60^2 Paris feet in one minute. One minute is a time unfamiliar in our experience for bodies to fall above the surface of the Earth. So Newton scaled the time of fall to one second. Conveniently, one second is 1/60th of a minute. Since the distance fallen varies with the square of time t, a body falling under the celestial force at the Earth's surface for one second would fall $1/60^2$ of 15.0934 x 60^2 Paris feet: that is, 15.0934 Paris feet. This matches well how bodies fall on the surface of the Earth under gravity, as measured by experiments on pendula. Newton ([1726] 1962, 408) concluded:

> And therefore the force by which the Moon is retained in its orbit becomes, at the very surface of the Earth, equal to the force of gravity which we observe in heavy bodies there. And therefore (by Rule 1 & 2) the force by which the Moon is retained in its orbit is that very same force which we commonly call gravity; for were gravity another force different from that, then bodies descending to the Earth with the joint impulse of both forces would fall with a double velocity. . . .

The case that Newton made here is a powerful one. In recollections recorded much later, he asserted that he had found the arguments of these first four propositions in 1666. He noted (1888, xviii) of the Moon test that

> At the same year [1666] I began to think of gravity extending to the orbit of the Moon, . . . and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the earth and found them answer pretty nearly.

3. The Inferences Summarized

The inference above can be summarized as follows:

Observed acceleration of fall of terrestrial bodies and the Moon.

(warrant) $H_{\text{inv. square}}$: The celestial force acting on the Moon is strengthened by an inverse square law with distance at the Earth's surface.

____(deduce)

Intermediate conclusion: Equality of accelerations at the Earth's surface due to gravity and the celestial force.

(warrant) Rules 1 and 2 of Newton's Rules of Reasoning in Philosophy

____

H_{identity}: Terrestrial gravitation and the lunar celestial force are the same.

The last step might seem to be superfluous. Newton found that the acceleration because of gravity and the celestial force match at the Earth's surface. Is that not enough to show the identity of the two forces? It is very close, but there is a loophole. It might just be that the force of gravity does not act on celestial matter such as comprises the Moon and that the celestial force does not act on ordinary, terrestrial matter. Newton closed the gap with the rules of reasoning that he had declared earlier in *Principia* ([1726] 1962). The relevant idea is that we are to assign the same cause to the same effect. I will not pursue this use of the rules further. In Chapter 6, "Simplicity," of *The Material Theory* *of Induction* (Norton 2021), I described my discomfort with the rules and indicated how they can be replaced in this case by a simple material fact: that the matter of the Moon would behave like terrestrial matter were it brought to the Earth's surface. What results is the simpler inference:

Observed acceleration of fall of terrestrial bodies and the Moon.

(warrant) $H_{\text{inv. square}}$: The celestial force acting on the Moon is strengthened by an inverse square law with distance at the Earth's surface.

____(deduce)

Intermediate conclusion: Equality of accelerations at the Earth's surface due to gravity and the celestial force.

(warrant) Terrestrial and lunar matter respond to the same forces.

____(deduce)

H_{identity}: Terrestrial gravitation and the lunar celestial force are the same.

For my purposes here, what matters is that the inverse square law, $H_{\text{inv. square}}$, is used as part of the inference to the identity result, H_{identity}. This usage forms half of the arch shown in Figure 8.1.

There is a second inference here that Newton did not make explicit. He inferred that the celestial force is governed by an inverse square law in other parts of the solar system. But how did he know that this inverse square dependence on distance would continue to hold when he moved out of the celestial realm down to the terrestrial realm? It is striking that the inference sketched above works so well. That the two forces "answer pretty nearly," as Newton remarked, gives one confidence that the inverse square law, introduced as a hypothesis above, is also supported by the successful outcome. Perhaps this was why Newton reported the agreement as a memorable phase in his discovery of universal gravitation. Although not given explicitly by Newton, we can summarize this naturally suggested argument as follows:

Observed acceleration of fall of terrestrial bodies and the Moon.

(warrant) $H_{identity}$: Terrestrial gravitation and the lunar celestial force are the same.

———————————————————————————————(deduce)

Intermediate conclusion: Celestial/gravitational accelerations at the Earth's surface and the Moon's orbit are in the ratio of an inverse square of distances to the Earth's center.

(warrant) Terrestrial and lunar matter respond to the same forces.

———————————————————————————————(deduce)

$H_{inv.\ square}$: The celestial force acting on the Moon is strengthened by an inverse square law with distance at the Earth's surface.

This second inference forms the second half of the relations of support displayed in Figure 8.1.

For my purposes here, we have two inferences each of whose conclusions is used as a warrant in the argument for the other. The corresponding arch can be drawn as follows.

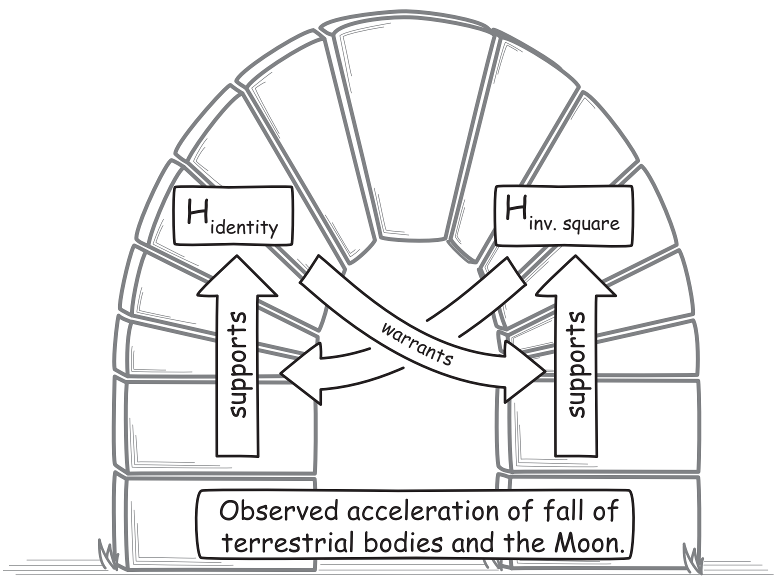

Figure 8.1. The arch for the Moon test

Although the component relations of support are deductive, the combined result is that the observed accelerations provide inductive support for the two hypotheses:

> Observed acceleration of fall of terrestrial bodies and the Moon.
>
> ____(induction)
>
> $H_{identity}$: Terrestrial gravitation and the lunar celestial force are the same.
>
> $H_{inv.\ square}$: The celestial force acting on the Moon is strengthened by an inverse square law with distance at the Earth's surface.

4. Elliptical Orbits and the Inverse Square Law

The next pair of mutually supporting propositions asserts that the planets move along elliptical orbits and that their motion is governed by an inverse square law of gravity. Planetary astronomy poses a curve-fitting problem. We have many observed positions of the planets. Which curve do we fit to them to recover their orbits? Prior to Newton, Kepler had found that elliptical orbits could be fitted to the observed positions of the planets. This result came to be known later as "Kepler's second law." It is called that, for example, in Maxwell's *Matter and Motion* (1894, 110). From it, one can infer that each planet is attracted to the Sun by a force that varies inversely with the square of distance from the Sun as the planet moves through its orbit. That an elliptical motion is associated with this inverse square law is an early result proved by Newton in Book I of *Principia* ([1726] 1962, Proposition XI, Problem VI). Maxwell uses this result to infer from the elliptical motions of the planets to the inverse square law of gravity:

> Hence the acceleration of the planet is in the direction of the sun, and is inversely as the square of the distance from the sun. This, therefore, is the law according to which the attraction of the sun on a planet varies as the planet moves in its orbit and alters its distance from the sun. (112)

That is, we have the following inference:

Observed positions of the planets.

(warrant) H_{ellipses}: The planets move in their specific elliptical orbits.

____(deduce)

$H_{\text{inv. square}}$: The planets are attracted to the Sun by a force that varies with the inverse square of distance.

Newton himself, however, was more circumspect. This relation of support is straightforward only insofar as we assume that the fit of an ellipse to the observed motions is exact. Newton knew that it is not exact, so he did not offer Maxwell's inference in his *Principia*. That an elliptical motion is governed by an inverse square law of force is merely reported as a theorem of mathematics.

In its place, Newton ([1726] 1962) offered an inverted relation of support. The pertinent discussion comes later in Book III in his Proposition XIII, Theorem XIII. At this stage in the development, Newton had already inferred the inverse square law of gravity from other phenomena. He would now infer from the inverse square law to the elliptical motions of the planets. Noting the inversion explicitly, he wrote that

> Now that we know the principles on which they [the motions of the planets] depend, from these principles we deduce the motions of the heavens *a priori*. Because the weights of the planets towards the sun are inversely as the squares of their distances from the sun's centre, if the sun were at rest, and the other planets did not act one upon another, their orbits would be ellipses, having the sun in their common focus. . . . (420–21)

Newton offered here a relation of support that inverts the one given above by Maxwell:

Observed positions of the planets.

(warrant) $H_{\text{inv. square}}$: The planets are attracted to the Sun by a force that varies with the inverse square of distance.

____(deduce)

H_{ellipses}: The planets move in their specific elliptical orbits.

The observed positions of the planets are still needed as a premise in the inference since an inverse square law of attraction from the Sun is also compatible with parabolic and hyperbolic trajectories. They are ruled out by the periodic motion of the planets. Then specific positions of the planets at specific times are needed to recover the specific ellipse that is the orbit of each planet.

Newton's inference, however, is qualified by an idealization indicated in his remark above "if the sun were at rest, and the other planets did not act one upon another." The orbits of the planets are not exactly elliptical because of perturbations from the gravitational attraction of the other planets. These deviations are generally negligible at the level of accuracy of Newton's analysis. However, a noticeable perturbation was produced by the massive planet Jupiter acting on the motion of Saturn.[1] It is greatest when the two planets are nearest each other: that is, when they are in conjunction. "And hence arises," Newton concluded, "a perturbation of the orbit of Saturn in every conjunction of this planet so sensible, that astronomers are puzzled with it" ([1726] 1962, 421).

5. The Exactness of the Inverse Square Law

Newton ([1726] 1962) did not explicitly incorporate the inference from the elliptical orbits of the planets to the inverse square law in the carefully developed sequence of propositions in Book III of *Principia*. However, an important step in that sequence was something close to this inference. It concerned the inverse square law of gravity. How did Newton know that the correct law is exactly an inverse square law? Might a similar law work as well or even better? Does gravity conform to the inverse square law only as an approximation? Perhaps the force varies with distance r according to $1/r^{2+\delta}$, where δ is some small number close to zero?

In one of the most brilliant analyses of his *Principia*, Newton showed that we have strong evidence for the force of attraction conforming exactly with the inverse square law. Under such a law, Newton had shown, the unperturbed planets move along elliptical paths fixed in space. The aphelion of each

1 Less noticeable, Newton reported, are the perturbations in Jupiter's motion because of the attraction of Saturn. He reported other perturbations as "yet far less" ([1726] 1962, 422). The exception was the sensible disturbance to the orbit of the Earth because of the Moon.

planet — the point of greatest distance from the Sun — will be fixed in space, and the planet will return to it after a complete circuit of 360° around the Sun. The major axis of the ellipse, the line of the apsides connecting aphelion and perihelion, will be correspondingly fixed.

This fixity would be lost, Newton now showed, if the law differed from an inverse XLV square law. In Proposition XLV, Corollary 1, of Book I, he considered the case of bodies orbiting in near-circular orbits. He showed that, if the law of attraction differed from an inverse square law, then a planet would not return to its aphelion after a circuit of 360° around the Sun. It would need to complete more or less of the circuit according to how much the force deviated from an inverse square law. That is, for a $1/r^{2+\delta}$ force law, the planet would return to its aphelion after passing $360^{\circ}/\sqrt{(1-\delta)}$. The result was remarkably robust, holding even when the deviation from the inverse square law δ was not small.

Since the planets do move in near-circular orbits, Newton could apply his result to the motions of the planets. If we set aside known perturbations, then the planets do trace fixed elliptical orbits, returning to their aphelia after a 360° circuit around the Sun. Newton could conclude with satisfaction in Book III, Proposition II, Theorem II that

> [The inverse square law] is, with great accuracy, demonstrable from the quiescence of the aphelion points; for a very small aberration from the proportion according to the inverse square0 law of the distances would (by Cor. 1, Prop. XLV, Book I) produce a motion of the apsides sensible enough in every single revolution, and in many of them enormously great. ([1726] 1962, 406)

In summary form, this argument is a version of Maxwell's argument since it infers from a property of the elliptical orbits of the planets to the exact inverse square law of gravity:

Observed positions of the planets.

(warrant) H_{ellipses}: The planets move in their specific elliptical orbits.

Newton's Proposition XLV, Corollary 1, Book I.

____(deduce)

$H_{\text{inv. square}}$: The planets are attracted to the Sun by a force that varies with the inverse square of distance.

The overall structure of the relations of support displayed here is of the two hypotheses accruing support from the observed positions of the planets over time. Although the two component inferences are deductive, the combined relations of support are inductive and can be summarized as

Observed positions of the planets.

____ (induction)

H_{ellipses}: The planets move in their specific elliptical orbits.

$H_{\text{inv. square}}$: The planets are attracted to the Sun by a force that varies with the inverse square of distance.

In broad strokes, the relations of support recounted here in Sections 4 and 5 are between the two hypotheses $H_{\text{inv. square}}$ and H_{ellipses}. They enter into the mutual relations of support pictured in the arch analogy of Figure 8.2.

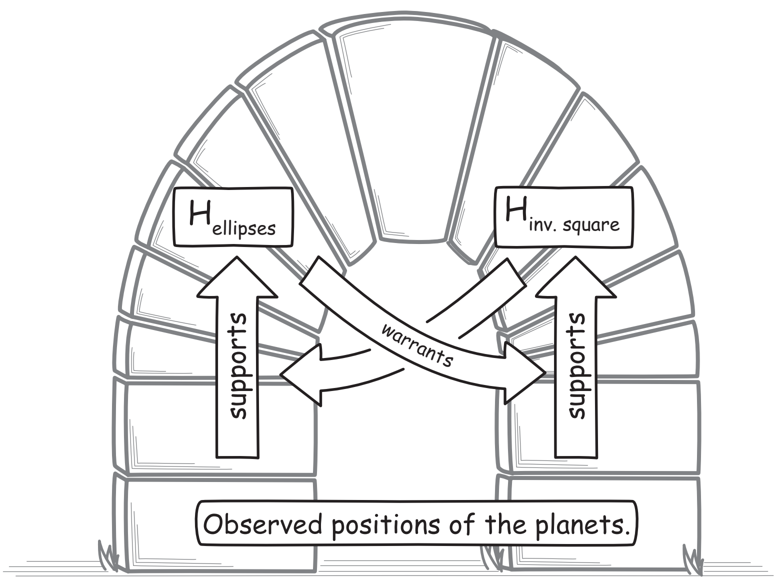

Figure 8.2. Elliptical orbits and the inverse square law

6. Conclusion

We have seen here two pairs of propositions in Newton's *Principia* ([1726] 1962) that mutually support one another. A close reading of his text is likely to reveal more. A natural candidate is Kepler's harmonic rule that relates the period and mean radii of planetary and lunar orbits: $(\text{period})^2$ is directly proportional to $(\text{radius})^3$. Newton infers from this harmonic rule to his inverse square law. We now routinely invert the inference and infer from the inverse square law to the harmonic law.

Such inversions are encouraged by a development common in maturing theories. We are inclined initially to infer from the elliptical orbits of the planets to the inverse square law of attraction, for the elliptical orbits are closer to observations. As the theory matures, we find multiple supports for the inverse square law. We also recognize that Newton's fully elaborated system corrects the simple statement that the planets move in ellipses, for in some cases the perturbing effects of other celestial bodies move them away from their ellipses. Then it becomes more natural to invert the relation of support and see the inverse square law as supporting a corrected version of the original observations of elliptical orbits.

Another example of this inversion is found in the role of atomic spectra in the foundation of quantum theory, as related in Chapter 9, "Mutually Supporting Evidence in Atomic Spectra." Ritz's combination principle supports the discrete energy levels of Bohr's 1913 theory of the atom and thus the quantum theory that developed from it. The developed quantum theory, however, entails a version of the Ritz principle, corrected by selection rules. This complication indicates the inverted relation of support.

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