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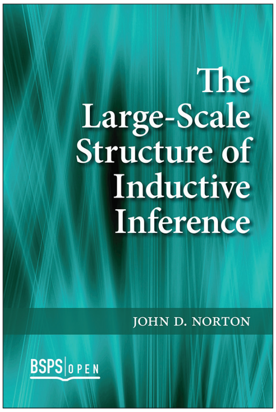

THE LARGE-SCALE STRUCTURE OF INDUCTIVE INFERENCE

John D. Norton

ISBN 978-1-77385-541-7

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The Use of Hypotheses in Determining Distances in Our Planetary System

1. Introduction

How distant from us are our nearest neighbors in space: the Moon, the Sun, and the planets?[1] This basic problem of astronomy proved to be a most challenging one that exercised astronomers from antiquity to as late as the nineteenth century. It provides a revealing case study of how hypotheses are used to extend the otherwise limited inductive reach of evidence.

One might expect that these distances could be determined by simple measurement, much as a terrestrial surveyor can determine the location and height of an inaccessible mountain peak. However, distances even to our closest body, the Moon, are so great that they present formidable challenges. Accurate triangulation of great distances requires extremely accurate angular measurements that were mostly beyond ancient astronomers, except perhaps for the closest body, the Moon. Even then, the ancient astronomers needed to await the opportunities provided by solar and lunar eclipses to break otherwise fatal evidential circles. The difficulty of making precise enough measurements meant that these methods were able to estimate distances only to the Moon and, so some extent, the Sun. These early efforts are described in Sections 2, 3, and 4 below.

The introduction of telescopes to astronomy in the seventeenth century made possible more accurate angular measurements. However, measurements of distance by means of triangulation, or parallax, as it is called in the

1 I thank Bernard Goldstein for helpful comments on an earlier draft.

astronomical literature, were limited at best to our closest planets, Mars and Venus. In Section 5, I recount the seventeenth-century measurement of the parallax of Mars, and in Section 6 I recount the eighteenth-century observations of the transits of Venus across the face of the Sun.

We find from all of these efforts that triangulation by itself is unable to provide much. This remains true even with careful telescopic measurements and a willingness to sail to distant parts of the globe to make them. The eighteenth-century measurements of the transits of Venus, by themselves, gave only angular displacements. Something more was needed if they were to deliver the distances to Venus and the Sun.

That essential extra was provided by hypotheses about the configuration of these celestial bodies. These hypotheses could extend the inductive reach of the few measurements available, and determinations of the distances to all of the celestial bodies mentioned became possible. This approach had been used from the first moments of ancient Greek astronomy and remained the primary approach used to the end of the nineteenth century. In the following sections, I review three different types of hypotheses used: Pythagorean and Platonic harmonies (Section 8), Ptolemy's *Planetary Hypotheses* (Section 9), and Copernicus' hypothesis of a heliocentric planetary system (Sections 10 and 11).

Examination of these three different hypothetical supplements gives us an opportunity to see how the hypotheses were used and should be used. The use of a hypothetical supplement takes on an evidential debt that must be discharged by finding independent evidence for the hypotheses. Only then have the results of the investigation been given proper inductive support. The need to discharge this debt is underscored by the fact that each hypothetical supplement reviewed leads to a different system of distances. Further evidence for the harmonic and Ptolemaic hypotheses was not secured, and they were discarded. The Copernican hypothesis, however, accrued considerable support. The most important was Newton's discovery of a mechanics that gave a dynamical foundation for the motions hypothesized in heliocentric astronomy.

What resulted was the edifice of classical mechanics. It combined astronomy and celestial and terrestrial mechanics in a single system, in which each part provided evidential support for the others. This crossing over of relations of inductive support is illustrated in the particular case of Kepler's third law and the inverse square law of gravity. Each, as I show in Section 12, provides inductive support for the other.

This reliance on hypotheses to enable the determination of distances within the planetary system persisted up to the nineteenth century, the latest extent of the history reviewed here. With the twentieth century, direct measurements of distances to celestial bodies became possible through laser and radar ranging.

2. An Evidential Circle: The Distances and Sizes of the Moon and Sun

How distant from us is the Moon? To appreciate just how formidable a question it was for ancient astronomers to answer, consider the majestic splendor of a full Moon rising over the eastern horizon at sunset. It is easy to imagine that the Moon is small and rises from a nearby place just over the horizon. That misapprehension is soon dispelled.[2] A house on a distant hill becomes larger as we approach it. But the Moon does not. No matter how far east we might venture, we see the Moon of the same size rising. Our eastward travels, from horizon to horizon, do not perceptibly diminish the distance to the Moon. We then realize that it is much more distant than we first thought. That means that it must be much larger than we first thought. How much larger is it?

That question leads to the first evidential circle. The disk of the full Moon fills about half a degree in our visual field. If we knew the size of the Moon, then we could calculate its distance by simple geometry. But if it were two, three, or four times larger, then it must be two, three, or four times more distant. As Figure 12.1 shows, many pairs of distances and sizes yield the same angular size in our visual field of half a degree. We cannot know the distance to the Moon until we know its size. But we cannot know its size until we know its distance.

2 This misapprehension is compounded by the "Moon illusion," in which it appears larger when near the horizon, although its measurable angular size is unchanged.

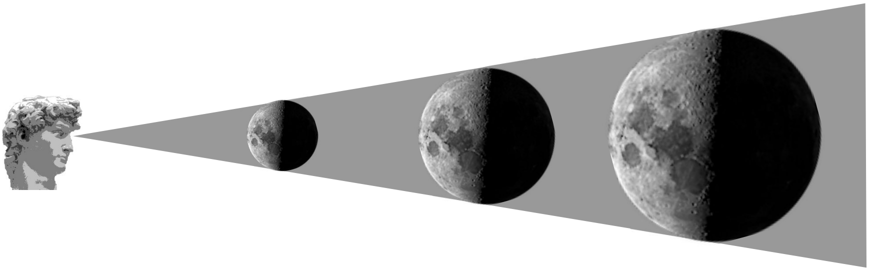

Figure 12.1. Many size-distance pairs for the Moon yield half a degree of angular size

All that these simple observations tell us is that the distance to the Moon must be large but otherwise leave it undetermined.

What of the relative distances of the Sun and Moon? We observe that the Sun has about the same angular size as the Moon of about half a degree. This equality is most easily learned from eclipses of the Sun. Then the Moon aligns with the Sun and almost perfectly obstructs it. Sometimes the Moon blocks out the Sun completely. Sometimes there is an "annular" eclipse in which the Moon blocks out the Sun, except for a thin annular ring of the Sun's surface encircling the Moon.

That the Moon eclipses the Sun shows that the Moon must be closer to us than the Sun. Are they roughly the same distance from us? If they are the same size, then they must be roughly the same distance from us. But if the Sun is two, three, or four times larger than the Moon, then by simple geometry the Sun must be two, three, or four times more distant from us than the Moon. As Figure 12.2 shows, we cannot know which until we know the true size ratio of the Sun to the Moon. But we cannot know that ratio until we know the ratio of the distances. We are trapped once again in an evidential circle.

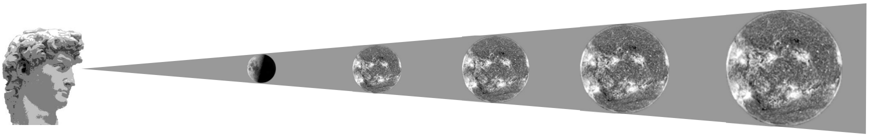

Figure 12.2. Many possible ratios of distances to the Sun and Moon

3. Aristarchus: Breaking the Evidential Circles

Both circles can be broken if we expand the evidence considered and are ingenious enough to do it in just the right way. This was the principal content of a remarkable document authored by Aristarchus of Samos, who lived roughly from 310 to 230 BCE. The work is presented in Greek and English translation in Heath (1913) under the title "Aristarchus on the Sizes and Distances of the Sun and Moon." Aristarchus breaks the evidential circle with two expansions of the evidence brought to bear. First, he introduces the angular positions of the Sun and Moon when the latter is precisely half illuminated: that is, at "dichotomy." Second, he introduces the behavior of the Moon during an eclipse of it, when it passes through the Earth's shadow.

When the Moon is exactly half full, the Sun, Earth, and Moon form a right-angled triangle, with the right angle at the Moon. The angle at the Earth is recoverable as the observable angular separation of the Sun and Moon. The shape of the triangle, shown in Figure 12.3, is thereby fixed, and the ratio of the Earth-Sun to Earth-Moon distance can be read from it.

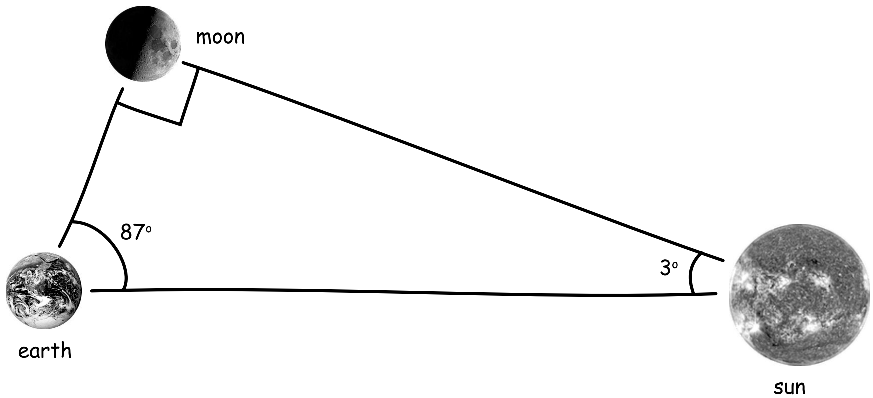

Figure 12.3. The Earth, Moon, and Sun at lunar dichotomy (not drawn to scale)

All that is needed is the angular separation of the Sun and Moon at dichotomy, as seen from the Earth. That is provided by the fourth of six hypotheses announced by Aristarchus (as given in Heath 1913, 353):

> That, when the moon appears to us halved, its distance from the sun is then less than a quadrant by one-thirtieth of a quadrant.

Since a quadrant is 90°, Aristarchus reports here that the angular separation of the Sun and Moon is 87°. After some analysis, he arrives at a ratio for the Earth-Sun to Earth-Moon distance that lies between 18:1 and 20:1.[3]

The method is ingenious and correct. However, it required the unattainable: an accurate measurement of the angular separation of the Sun and Moon at the moment of dichotomy. Aristarchus greatly underestimated the true ratio of 389:1.[4]

As far as the ratios of distances were concerned, Aristarchus had broken the evidential circle. He had established, he believed, the ratio of distances

3 Aristarchus did not have tables of tangents to consult, which now makes our computation trivial. The exact result is tan 87 = 19.08.

4 Dreyer (1953, 136) diagnoses the error as follows: "The method, though theoretically correct, is not practical, as the moment when the moon is half illuminated cannot be determined accurately. The angle of 'dichotomy' is in reality 89° 50' instead of 87°."

to the Sun and Moon. He could then infer directly to the ratio of the diameters of the Sun and Moon. It must be the same. It must also lie between 18:1 and 20:1.

Aristarchus then turned to determine not just the ratios of the distances to the Sun and Moon but also their individual values. They were expressed as ratios with the diameter of the Earth, whose value was then known well enough. Heath (1913, 399) presumes that Aristarchus did as Archimedes did and accepted Dicaearchus' estimate of a circumference of 300,000 stades. Eratosthenes' famous measurement of the Earth's size came later. Aristarchus realized that these individual distances could be recovered from phenomena observable at the time of an eclipse of the Moon. To determine these individual distances, he introduced a decisive new datum concerning an eclipse of the Moon (Heath 1913, 353):

> That the breadth of the [Earth's] shadow is [that] of two moons.

That is, as the Moon passes through the umbra, the conically shaped, full shadow of the Earth, the Moon's diameter is just half that of the umbra, as Figure 12.4 shows. What results is a complicated geometric figure that has been reproduced in many old manuscripts and modern treatises and is not drawn to scale in Figure 12.4. The lower figure depicts the essential geometry and is reproduced from Heath's (1913, 330) analysis.[5]

5 This work is in the public domain.

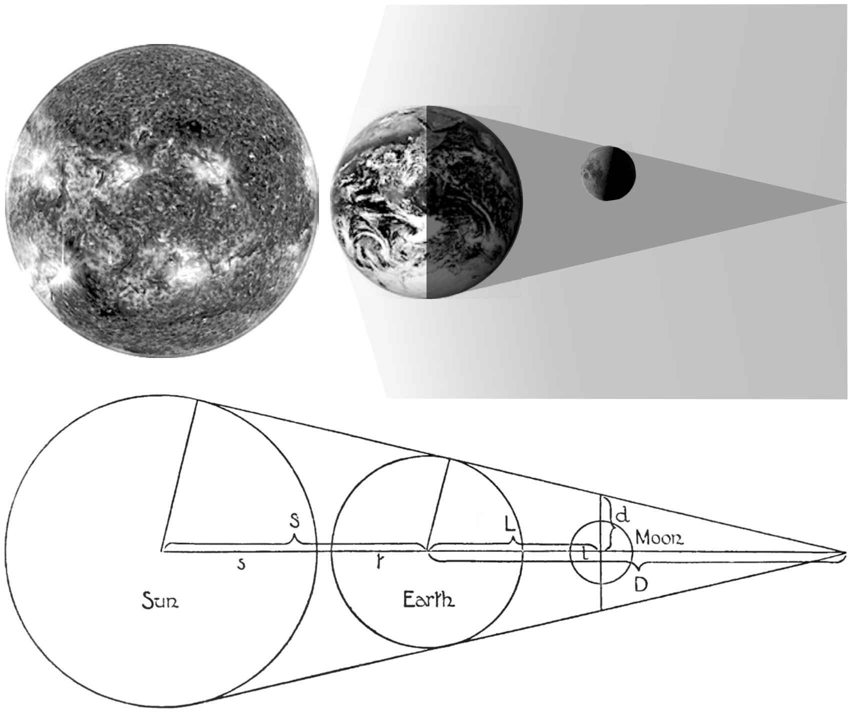

Figure 12.4. Aristarchus' figure for determining the distances to the Sun and Moon

Since the ratio of the diameters of the Sun *s* and Moon *l* are known, as are the ratios of the distances from the Earth to Sun *S* and Earth to Moon *L*, it turns out that the geometry of Aristarchus' figure is fixed. This might not be obvious from inspection of the figure, and it takes some calculations to determine it. Since they are tedious and not especially illuminating, I refer the reader to Heath's (1913, 330–31) reconstruction. Aristarchus arrived at a diameter for the Sun as a ratio of the Earth's diameter that lies between 19/3 and 43/6 and a diameter for the Moon as a ratio of the Earth's diameter that lies between 19/60 and 43/108. Once again, with the diameters of the Sun and Moon determined, it was a simple matter to determine the distances to the Sun and Moon from the known angular size of each as seen from the Earth.

The actual numbers reported by Aristarchus are quite far from the actual ratios in our Sun-Moon-Earth system.[6] His calculations depended on his earlier underestimate of the ratio *S/L* of the Earth-Sun and Moon-Sun distances. They were compounded by his taking erroneously that the angular size of the Moon is 2°, whereas Archimedes in the *Sand-Reckoner* had attributed the correct $1/2^{\circ}$ to Aristarchus.[7]

Although Aristarchus' final numbers differ greatly from the actual values, his methods were correct and ingenious, marred only by the need for an impractical datum and a curious error in estimating the Moon's size. Van Helden (1985, 7) singles out Aristarchus' second Moon eclipse technique as

> . . . a method that, when fully developed by Hipparchus and Ptolemy, was to be the centerpiece of all determinations of absolute celestial distances until the seventeenth century.

4. Measurements of Parallax

The methods reviewed so far require that the disks of the Sun and Moon be discernible. As long as astronomers use only naked-eye methods, they cannot determine distances to the planets, for optical instruments are needed to resolve their disks. There is a general method that, in principle, is capable of determining the distance to any celestial object visible from the Earth. That is the measurement of its parallax. It is the difference in direction of some object as seen from different places on the Earth. Measuring it requires observations to be taken at two different places at the same time. For the case of a rotating Earth, parallax can also be measured from one position on the Earth when the rotation moves that position to another location in space.

Horizontal parallax uses the Earth's radius as the baseline for measurement.[8] Figure 12.5 shows an observer at *A* on the Earth's surface who finds the object at *P* to be at its zenith: that is, directly overhead. A second observer at *B*, located at a distance of one-quarter of the Earth's circumference, finds

6 Aristarchus' ratio for the Sun is 6.3 to 7.2, where the modern figure is 109. His ratio for the Moon is 0.31 to 0.40, where the modern figure is 0.27.

7 See Heath (1913, 311–14) for an analysis of this curious error.

8 It is distinguished from annual parallax, in which the radius of the Earth's annual orbit around the Sun is used as the baseline for measurement. It is applied when considering distances to stars.

the object to have just dipped below the horizon. If we draw *BC* parallel to *AP*, then the bearings of the object at *P* differ for these two observers by the angle of parallax, *CBP*. This angle is equal to the angle *BPA*, the angle subtended by the Earth's radius from *P*.

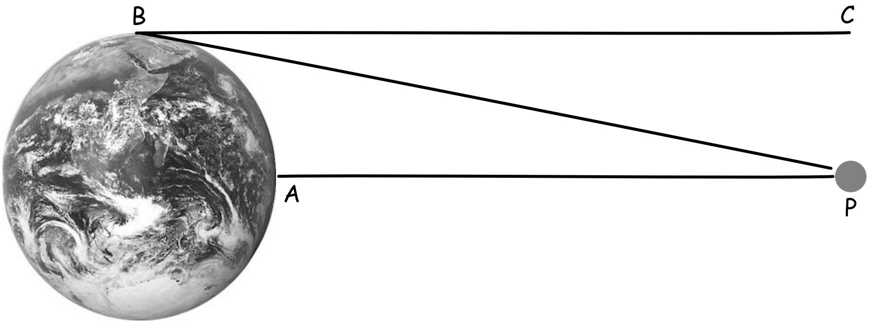

Figure 12.5. Horizontal parallax

This angle is called "horizontal parallax" since the name reflects *B*'s observing *P* on the horizon. For a distant object, the angle is small[9] and related inversely to the distance to the object by

distance = radius of the Earth / horizontal parallax in radians

In practice, horizontal parallax is not measured directly. A smaller displacement on the Earth's surface is used and horizontal parallax inferred from it.

Once again, eclipses provided opportunities for potentially informative measurements. An eclipse of the Sun will be total when seen from one part of the Earth's surface yet only partial when seen from another part. Encoded in this difference is a measure of the parallax of the Moon. Hipparchus and Ptolemy after him applied this approach to records of lunar eclipses to estimate lunar parallax.[10] Although the method is correct in principle, its successful application is difficult because of the need to measure angles precisely. The Moon's parallax of about 1° is the largest for celestial objects. Others are

9 The figure greatly exaggerates the angle. For the Moon, the horizontal parallax varies about roughly a degree. For the Sun, it is about 8.8 seconds of arc: that is, 2.4 thousandths of a degree.

10 For details, see Van Helden (1985, 10–19).

dauntingly smaller. Measurement of the tiny solar parallax of 8.8 seconds of arc was beyond the reach of the ancient astronomers.

5. The Parallax of Mars

The difficulty of measuring tiny parallactic angles was only overcome centuries later when telescopic observations were possible. Even then, the approach was indirect. The Earth-Sun distance was the most sought once heliocentric Copernican astronomy became established. It determined, as we shall see below, all of the other distances. However, direct measurement of the parallax of the Sun remained beyond the astronomers' reach, if only because the brilliance of the Sun precluded direct observations locating it against the stellar background. Instead, it proved to be feasible to determine the parallax of Mars and, using the known ratio of sizes of the orbits of Mars and the Earth, then compute the Earth-Sun distance.

The best known of these measurements of parallax from the seventeenth century is Cassini and Richer's measurement of the parallax of Mars in 1672 using simultaneous measurements of the position of Mars from France and Cayenne in South America. The opportunity for the measurements was an opposition of Mars to the Sun. That meant that Mars was making one of its closest approaches to the Earth and thus susceptible to the most accurate measurements. Their efforts yielded the parallax of Mars at this time in its orbit and thus its distance from the Earth. Using the then known ratio of the sizes of the orbits of the Earth and Mars, the crucial Earth-Sun distance could be estimated. Cassini and Richer arrived at an Earth-Sun distance of 87,000,000 miles, comfortably close to the modern value of about 93,000,000 miles.[11] Both Berry (1898, 205–09) and Van Helden (1985, Chapter 12) emphasize that the closeness of these numbers is less impressive once one recognizes the large margin of error associated with the Cassini and Richer result.

6. The Transits of Venus

The ancient astronomers had found solar eclipses to afford opportunities to determine the parallax of the Moon. These eclipses arise when the Moon passes exactly between the Earth and the Sun. An analogous circumstance

11 See Long (1742, 290, 292) for an early account.

arises when the planet Venus passes exactly between the Earth and the Sun. Since Venus is so much farther away from the Earth than the Moon, the effect is much less dramatic. Venus appears telescopically as a tiny dot migrating over the surface of the Sun. If this "transit of Venus" is observed from different locations on the Earth's surface, then Venus will be seen to transit across the disk of the Sun in different locations on the disk.

The path of Venus traces a chord across the circular disk of the sun. Determining the length of the chord fixes its location on the disk. The longest chords are diameters of the circle; the shorter the chord, the farther it is from a diameter. The most accurate way to estimate the difference in chord lengths was to time how long the transit took, when viewed from different locations. Since a transit requires about six hours, accurate times of transit were well within the grasp of measurement of early clocks. The transit times reflect directly the chord lengths and thus reveal differences of location of the transits against the Sun's disk.

Observing a transit of Venus from different places on the Earth enabled the parallax of Venus and the Sun to be determined. Of the expeditions to observe the transit of Venus, the best known, especially to Australians, is that of Captain Cook, who sailed to Tahiti for this purpose in 1769. The measurements of the Cook expedition were compared with those taken in other locations, notably Lapland. The resulting parallax of the Sun was determined to be in the range of eight to nine seconds of arc, in agreement with the modern value of about 8.8 seconds of arc.[12] Subsequent transits were observed in 1874, 1882, and more recently in 2004 and 2012.

The calculation of the parallax of Venus and the Sun from these observations must correct for many factors. The highly simplified analysis in Figure 12.6 brings out the element most important for my purposes here.

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12 For accounts of the transits, associated measurements, and calculations, see Airy (1881, 144–60) and Newcomb (1892, 177–92). That these expeditions and measurements were of considerable popular interest in the nineteenth century is suggested by the publication of popular works such as Forbes (1874).

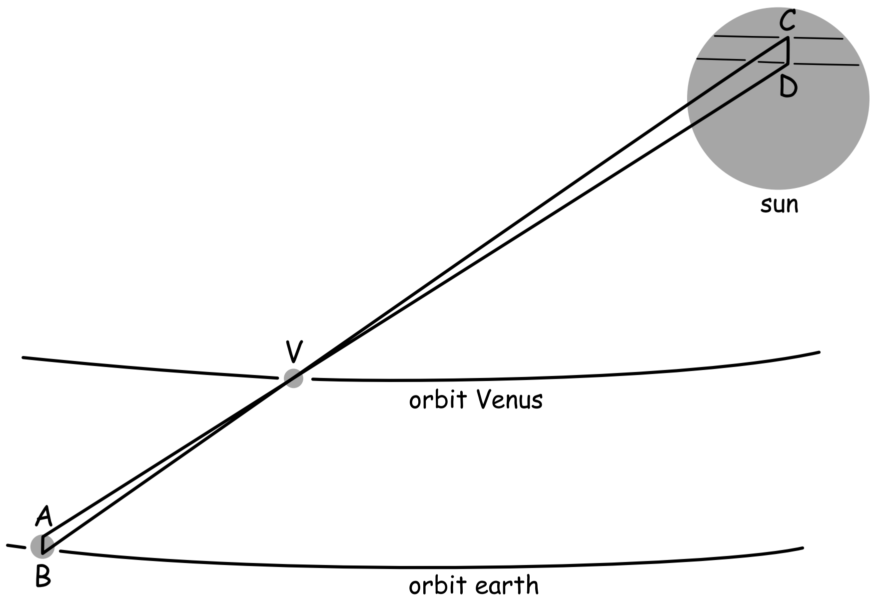

Figure 12.6. Transit of Venus; redrawn from Airy (1881, 153)

Points *A* and *B* are the locations of two observers on the Earth's surface. They are as widely separated as possible. *A* might be in the northern hemisphere. *B* might be in the southern hemisphere. The lines of sight *AVD* and *BVC* pass through Venus at *V* to different locations *D* and *C* on the Sun's disk. The distance *CD* is the separation between the two transit paths observed. If the absolute distance of *CD* can be determined, then it can be scaled up to give the absolute diameter of the Sun. Since the angular size of the Sun as seen from the Earth is readily measured, the distance to the Sun can be recovered.

Triangle *ABV* and *DCV* are similar. Thus, the distance sought, *CD*, can be found from the formula

$$CD = (DV / AV) \cdot AB$$

The distance *AB* is the known distance between the two observers on the Earth. The ratio *DV*/*AV* is determined by the ratios of the sizes of the planetary orbits. These last ratios were given by Copernican astronomy, as we shall see below.[13] Without knowledge of this ratio, we would be trapped once again in the familiar evidential circle. A small ratio *DV*/*AV* would lead to a small distance *CD* and a small Earth-Sun distance. A large ratio *DV*/*AV* would lead to a large distance *CD* and a large Earth-Sun distance. Some further datum, such as the absolute length *CD* itself, would be needed to break the circle.

7. The Need for Hypotheses

The efforts recounted above reveal the limits of simple geometric triangulation as a means of determining distances to bodies in our planetary system. This approach was able to arrive at a distance to the Moon and, when pressed to the extreme in the seventeenth century, a distance to Mars at its closest approach to the Earth. Even as late as the eighteenth and nineteenth centuries, these methods of triangulation had to be supplemented by further knowledge of the planetary system if their results were to be extended to a determination of the Earth-Sun distance. The seventeenth-century determination of the distance to Mars could be extended only to an estimate of the distance to the Sun by drawing from the known ratio of the sizes of the orbits of the Earth and Mars. The eighteenth-century and nineteenth-century observations of the transits of Venus were unable to return any absolute planetary distances until they were augmented by the known ratio of the sizes of the orbits of the Earth and Venus.

At the close of the nineteenth century, observations of the transit of Venus remained the best way to determine distances within the solar system. After a lengthy treatment of the transits of Venus, Simon Newcomb (1892, 192–99), then a leading authority in astronomy, added a discussion entitled "Other Methods of Determining the Sun's Distance, and Their Results." The promise of these "other methods" went unfulfilled. Newcomb could only say of them that ". . . at least two of which [methods] we may hope, ultimately, to attain a greater degree of accuracy than we can by measuring parallaxes" (192).[14]

13 Hipparchus' analogous determination of the parallax of the Moon at the time of a solar eclipse avoided the need for a corresponding ratio. Hipparchus could assume that the Sun was so much farther from us than the Moon that the Sun's rays arrived in parallel lines on the Earth.

14 How things change! Lasers, reflected off mirrors left on the Moon by manned and unmanned missions in the 1960s and early 1970s, now determine the distance to the Moon to within a few centimeters. We can now also use radar echoes to measure distances to the planets.

From the earliest times, the sort of supplement needed was already present as hypotheses of various types. Our histories of astronomy treat the early ones dismissively since most of these early supplements were in error. Since my concern here is not the correctness of the results but the appropriateness of the inductive strategies, we can arrive at a more favorable appraisal. Direct evidence, such as distance measurements by triangulation, can fail to give us the extent of the results sought, such as the distances to the Sun and distant planets. We can then conjecture or hypothesize those facts that would extend the inductive reach of the evidence available to us. This is an entirely responsible epistemic strategy as long as we remember that adopting a hypothesis takes on an inductive debt. It has to be discharged by further investigation that will provide independent inductive support for the hypothesis. Only then have we given the new results a solid foundation inductively in evidence.

8. Pythagorean and Platonic Harmonies

The Pythagorean and Platonic tradition in antiquity provided a rich if chaotic set of hypotheses concerning the distances to the celestial bodies. Their basis was a combination of ideas in musical harmony and simple arithmetic relations. In his creation myth, *Timaeus*, for example, Plato offers the following relative distances:

Moon	1
Sun	2
Venus	3
Mercury	4
Mars	8
Jupiter	9
Saturn	27

These ratios arise from interleaving the numbers of two geometric progressions: 1, 2, 4, 8 and 1, 3, 9, 27. They are just a small part of a rich collection of proposals.[15]

15 For a terse review, see Dreyer (1953, 62, 178–82).

If Plato's ratios are correct, then the inductive benefit is immediate. We need only determine the absolute distance to any one of these celestial bodies. Then the absolute distances to all of the others can be determined from the ratios. What would suffice is just one of the later determinations of the distance to the Moon by Aristarchus or Hipparchus.

It is easy for us now to dismiss these harmonic hypotheses as wild speculations.[16] They were indeed highly speculative. That they were likely incorrect would have been apparent to Aristarchus and Hipparchus. Plato located the Sun at twice the distance from the Earth as the Moon, but both Aristarchus and Hipparchus determined that the Sun is much more distant. That does not make Plato's conjectures epistemically irresponsible. Conjectures of some sort were the only way then available to advance the project of determining distances to celestial bodies beyond the distances accessible to measurement by triangulation. Might it just be that this particular implementation of the harmonic idea is flawed? Might further refinement produce a proposal that can survive independent scrutiny?

Johannes Kepler has unchallengeable credentials in astronomy. He believed that these harmonic ideas found their proper expression in the new heliocentric Copernican astronomy. His *Mysterium cosmographicum* of 1596 accounted for the number of planets and the ratios of planetary orbits by a celebrated geometric construction involving the five Platonic solids, nestled between spheres. The image from that work, shown here as Figure 12.7, has been widely reproduced.

16 Dreyer (1953, 181) writes that "in reality therefore we ought hardly to take the planetary intervals, as determined by the sphere-harmony, seriously; the whole doctrine is quite analogous to that of astrology, but is vastly more exalted in its conception than the latter, and it deserves honourable mention in the history of human progress."

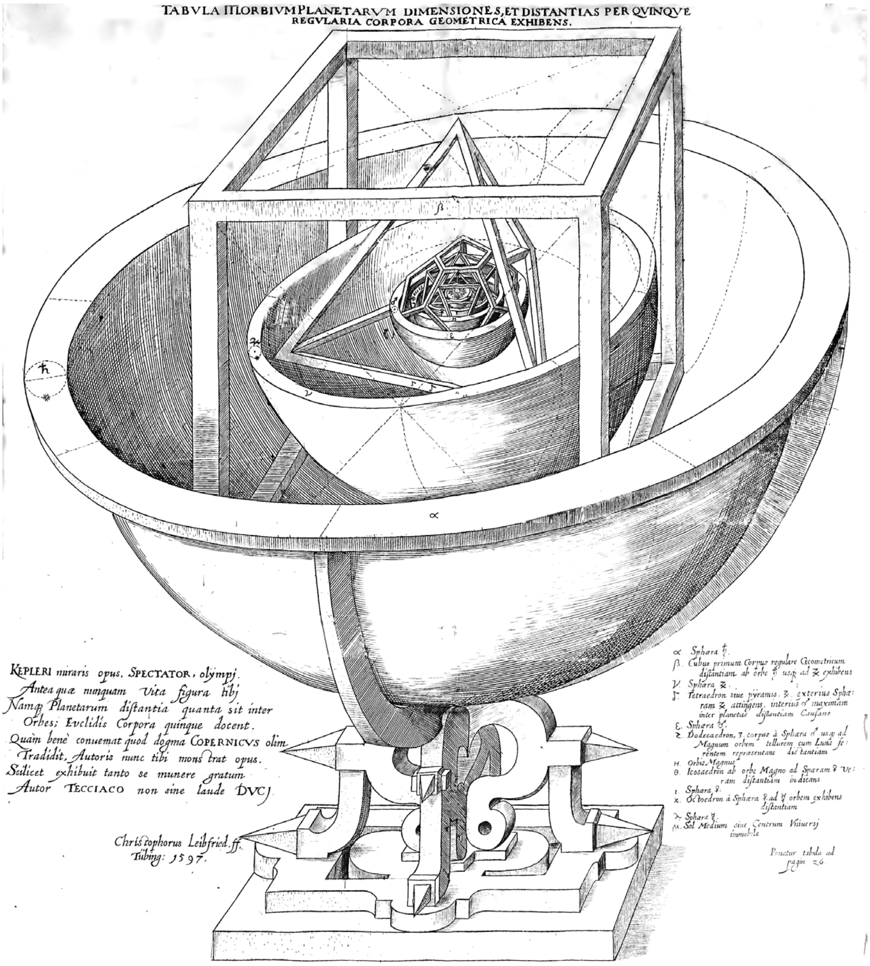

Figure 12.7. Kepler's construction

Kepler's *Harmonices mundi* of 1619 proceeded to find musical harmonies in planetary motions. Although we now dismiss these parts of Kepler's work as mistaken, they were part of a serious investigation. They were hypotheses that failed to find independent evidential support. Had they found such support, we would now be celebrating Kepler's prescience.

The tradition of seeking mathematical harmonies persists. In his Herbert Spencer lecture in 1933, an older Einstein revealed his conversion to a form of mathematical Platonism.[17] He wrote that

> Our experience hitherto justifies us in believing that nature is the realisation of the simplest conceivable mathematical ideas. I am convinced that we can discover by means of purely mathematical constructions the concepts and the laws connecting them with each other, which furnish the key to the understanding of natural phenomena. (1933, 274)

Lest there be any doubt that Einstein saw his formulation of these ideas as fulfilling the program initiated millennia ago in ancient Greece, he added

> But the creative principle resides in mathematics. In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed. (1933, 274)

9. Ptolemy's *Planetary Hypotheses*

The supreme expression of geocentric astronomy in antiquity was Ptolemy's second-century AD *Almagest*. It provides elaborate geometric constructions of the motions of the celestial bodies: the Moon, the Sun, and the planets. The constructions, however, were independent of the absolute size of the orbit of each body. Take, for example, the construction for Venus. This planet moves roughly with the Sun in its annual course around the heavens. But Venus is sometimes ahead of the Sun and sometimes behind it. This direct and retrograde motion was accounted for in Ptolemy's construction by attaching the planet to a rotating epicycle, as shown in Figure 12.8. The epicycle's center moves along a deferent circle such that this center remains aligned with the mean Sun. (The actual motion of the Sun deviates slightly from the mean motion.)

17 See Norton (2000) for an account of Einstein's conversion.

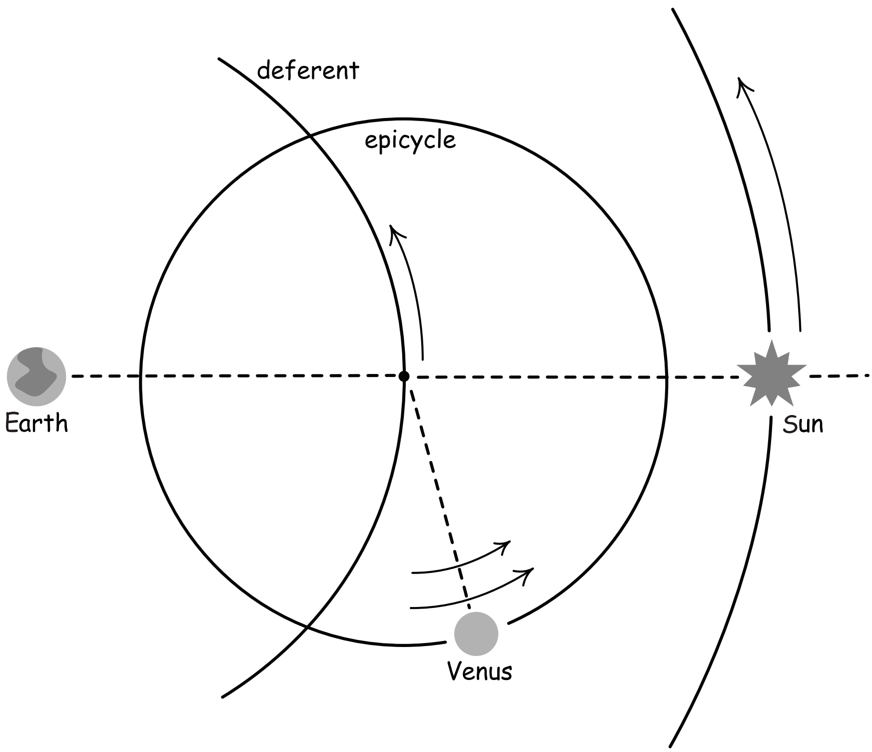

Figure 12.8. The Ptolemaic epicycle for Venus

The figure shows the motion of Venus drawn within that of the Sun. That is not needed to recover the retrograde motion of Venus. As long as the alignment of the center of the epicycle and the Sun is retained, the construction for Venus could be expanded so that its motion would be outside that of the Sun, Mars, Jupiter, or Saturn. The construction for each celestial body in Ptolemy's *Almagest* could be scaled up or down so that any order of the Sun and planets was possible.

The determination of the absolute sizes of these trajectories was taken up in a later work by Ptolemy, *Planetary Hypotheses*. The portions of it dealing with these distances have been lost in the extant Greek texts. Goldstein (1967) found them in a later Arabic translation, and his paper presents an English translation of the Arabic along with the original Arabic text.[18] In addition to the geocentric supposition, Ptolemy's analysis depended on two hypotheses:

Order. The celestial bodies increase in distance from the Earth in the order Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn.

Packing. The celestial bodies are packed together as closely as their geometrical constructions allow.

These hypotheses provided Ptolemy with the ratios of the distances to the celestial bodies. He could then combine these ratios with his estimate of the absolute distance to the Moon to recover the distances to all of the celestial bodies.

These hypotheses did not derive from considerations of musical and mathematical harmony. Rather, they rested on prosaic, physical considerations. To recover *Order*, we know that the Moon is closer to the Earth than the Sun and stars since the Moon eclipses them. The rest of the order was harder to pin down. The stars have the fastest motion in the Ptolemaic system, with Saturn, then Jupiter, and then Mars lagging successively more behind them. Assuming that proximity in speed reflects proximity in space, Ptolemy could conclude that Saturn is the closest to the stars; then comes Jupiter and then Mars. By this criterion, the Sun, Venus, and Mercury come next. However, the criterion could not give an order for them since their average motion against the stars was the same. Ptolemy settled on the order the Sun, then Venus closer to the Earth, and then Mercury closer still. He reasoned that the closeness of Mercury to the Moon was justified by the similarity of their eccentric motions and since the frequent retrograde motion of Mercury resembled the turbulent motions of the air above the Earth's surface. Similar reasoning placed Venus at the next distant position.

To establish the absolute distances to these celestial bodies, Ptolemy employed the fact that his constructions would take each body nearer to and farther from the Earth. The epicycle shown in Figure 12.8 does this, as does Ptolemy's use of eccentric circles: that is, circles whose centers are slightly displaced from the Earth. Ptolemy could determine from these constructions the ratio of the distances of closest approach to the Earth (perigee) and the farthest displacement (apogee). He now assumed (*Packing*) that all of the constructions were packed together as closely as the geometry allowed, without

18 For further analysis, see Van Helden (1985, 21–27).

the danger of any of the trajectories intersecting. That is, the apogee of the Moon will coincide with the perigee of Mercury, the apogee of Mercury will coincide with the perigee of Venus, and so on.

Ptolemy could only offer the physical plausibility of this packing assumption: “This arrangement,” he wrote, “is most plausible, for it is not conceivable that there be in Nature a vacuum, or any meaningless and useless thing” (quoted in Goldstein, 1967, 8). He could not have been so certain of the assumption, for he proceeded to allow that, if there are empty spaces, then the distances cannot be smaller than those that he had determined.

Starting with his value of 64 Earth radii for the apogee of the Moon, Ptolemy used the ratios of perigee to apogee to determine stepwise the distances to all of the celestial bodies. The perigee of Mercury is then 64 Earth radii. The ratio of perigee to apogee for Mercury is 34:88, so its apogee is at 64 x (88/34) = 166 Earth radii. Continuing these calculations leads to the results summarized in Table 12.1.[19]

Table 12.1. Ptolemy’s distances in units of Earth radii

	Perigee	**Apogee**	**Ratio**
Moon	33	64	33:64
Mercury	64	166	34:88
Venus	166	1,079	16:104
Sun	1,160	1,260	57.5:62.5
Mars	1,260	8,820	1:7
Jupiter	8,820	14,187	23:37
Saturn	14,187	19,865	5:7

Ptolemy encountered one discrepancy. His independent estimate of the perigee of the Sun is 1,160, which does not match the computed apogee of Venus of 1,079. He suggested that the discrepancy might derive merely from slight errors in the underlying observations. To continue, Ptolemy used the independently derived figure of 1,160 for the Sun’s perigee.

19 Ptolemy’s text delivers these results in a continuous narrative. This convenient tabular summary is provided by Van Helden (1985, 27). He notes that the value of the apogee of Jupiter of 14,187 is a small error of calculation and should be 14,189.

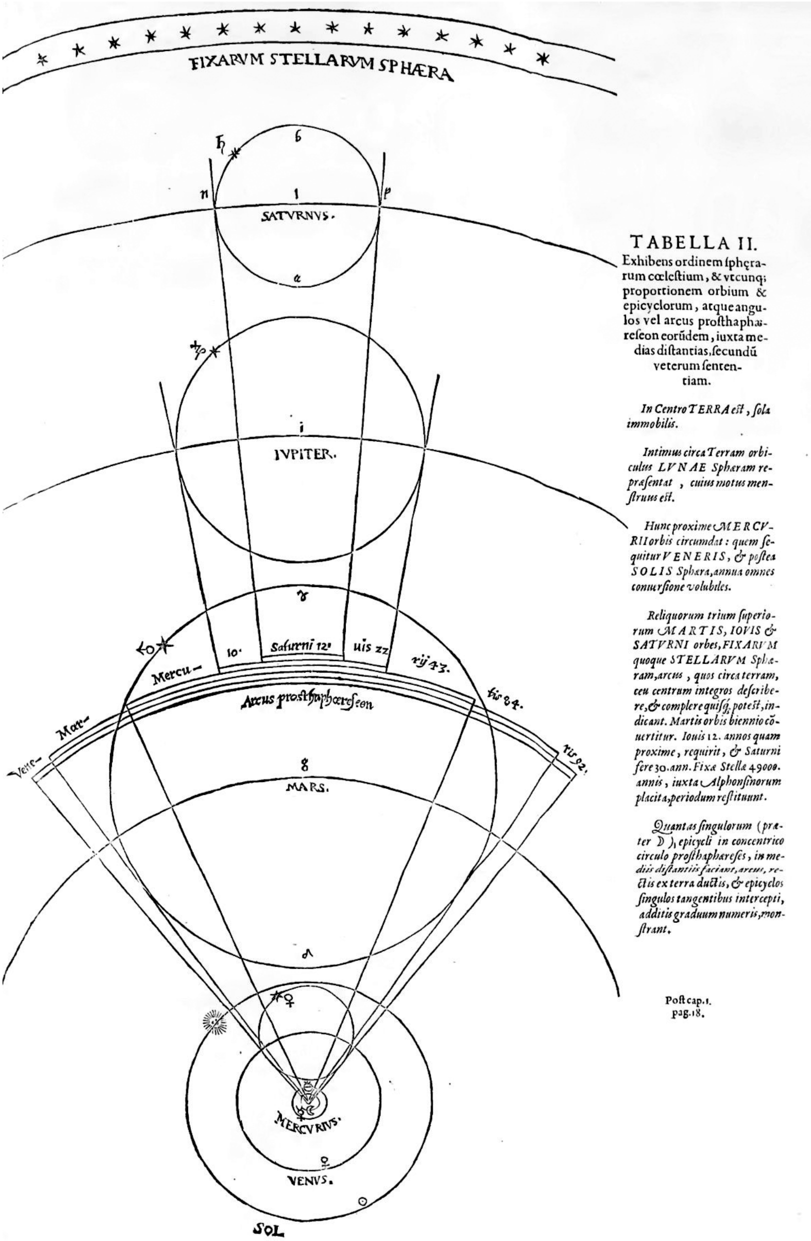

Figure 12.9. Kepler’s drawing of the Ptolemaic system

Kepler's *Mysterium cosmographicum* of 1596 has a figure (Figure 12.9 here) that includes all of these celestial bodies in Ptolemy's system, with their epicycles, drawn approximately to the scale set by the distances of Table 12.1.

In assuming the geocentric configuration of celestial bodies and in making the assumptions of order and packing, Ptolemy took on an inductive debt. Until it was discharged — that is, until independent evidence for the assumptions was found — the evidential case for his distances was incomplete. Ptolemy counted as evidence for his packing hypothesis the closeness of the two estimates of the distance to the Sun's perigee: the packing-derived estimate of 1,079 and the independent estimate of 1,160. Although encouraging, that closeness was not enough to discharge the inductive debt. Further independent support was needed. Although Ptolemy's system remained the authoritative system for over a millennium, that further independent support never came. His system was abandoned in favor of another whose inductive debts were discharged and with spectacular success.[20]

10. The Copernican Hypothesis

Nicolaus Copernicus' *On the Revolutions of the Heavenly Spheres* of 1543 is somewhat tame in purely astronomical terms. In the simplest concept, it merely rearranges the circles of Ptolemy's geocentric system in a more apposite way. It is in another sense Earth moving. That rearrangement sets the Earth into twofold motion: spinning on its axis and orbiting the sun.

This basic supposition of Copernican heliocentric astronomy was routinely known as the "Copernican hypothesis" or "hypotheses" in the sixteenth and seventeenth centuries. Moxon's (1665) *Tutor* offered the reader on its title page *an Explanation of the Copernican Hypothesis and Spheres*. Hooke (1674) uses the expression liberally. In the sixteenth century, the term "hypothesis" was tainted by Osiander's surreptitious insertion of an anonymous preface into Copernicus' 1543 work. Osiander reduced Copernicus' proposal to a mere convenience of calculation that did not reveal true causes. He wrote that "these hypotheses need not be true nor even probable. On the contrary, if they provide a calculus consistent with the observations, that alone is enough" (Dobrzycki 1978, xvi).

20 For a survey of the persistence of Ptolemy's packing hypothesis through to the time of Kepler in the sixteenth century, see Goldstein and Hon (2018).

Copernicus himself made little use of the term but did not seem to be averse to it. Conveniently, he did equate the term in its usage by the Greeks with "principles and assumptions" (Dobrzycki 1978, 7).[21] As far as I can see, the term "hypothesis" does not appear in his earlier draft manuscript "Commentariolus." However, the main proposals of his heliocentric astronomy are called "assumptions" (Rosen 1971, 58). Rheticus uses the term "hypothesis" freely in his preliminary accounting of Copernicus' proposal, *Narratio prima*, written prior to 1543.[22] He goes to some pains to defend the truth of the hypotheses that he identifies in Copernicus' system. His defense foreshadows the present notion of hypothetico-deductive confirmation: it is a mark of truth if a hypothesis has true consequences. Rheticus puts it this way: "Aristotle says: 'That which causes derivative truths to be true is most true'" (Rosen 1971, 142).[23] In this context, then, common use of the term "hypothesis" referred to an adventurous proposal. Contrary to Osiander's pessimism, its truth could be secured through argument and evidence, and it was thus secured as we moved from the sixteenth century to the seventeenth century.

For my purposes here, what matters is that adoption of Copernicus' heliocentric system proved to be the key step in expanding astronomers' capacity to determine the distances to celestial bodies. Ptolemy needed to add hypotheses, *Order* and *Packing*, to his geocentric constructions in order to fix the ratios of these distances. Copernicus needed no such additions to determine the ratios of the orbital sizes. His heliocentric constructions already fixed them.

The recovery of these ratios followed from how Copernicus' system reduced the number of independent assumptions needed compared with those required by Ptolemy. Consider, for example, Ptolemy's construction for Venus as shown in Figure 12.8. Copernicus realized that two motions in Ptolemy's system were really just one. That is, the annual motion of the center of the epicycle of Venus along the deferent and the annual motion of the Sun were not real motions at all. Rather, there was just the single annual motion of the Earth around a central point near the Sun and then around the Sun itself in

21 Copernicus writes that astronomy's "principles and assumptions" were "called 'hypotheses' by the Greeks."

22 Reproduced in translation in Rosen (1971).

23 There is an extensive secondary literature on Copernicus' attitude to hypotheses; see Rosen (1971, 22–33).

later developments of heliocentrism, such as by Kepler. If an observer on the Earth was unaware of its motion, then it would appear that both the Sun and Venus were orbiting the Earth. These two circles were just apparent motions arising from displacing the true motion of the Earth to Venus and the Sun.

To accommodate this realization, Copernicus rearranged the circles in Figure 12.8 to recover those in Figure 12.10. As shown at the top of the latter figure, the two circles of the deferent of Venus and the Sun were collapsed into a single circle, and that circle was transposed to become the orbit of the Earth around the Sun. The epicycle of Venus now became its true orbit, centered on the Sun.

This new heliocentric construction for Venus no longer admitted the arbitrary rescaling of planetary distances that troubled Ptolemy's system. The maximum elongation of Venus — the maximum angular distance that it strayed from the Sun — was about 45°. That fact of observation immediately fixed the ratio of the sizes of the orbits of Venus and the Earth. The line *EV* in Figure 12.11 traces the line of sight to Venus at its maximum elongation. Since *EV* is tangent to the circle of the orbit of Venus, *EVS* is a right angle. If we take the simplest case of the angle *EVS* equal to 45°, then the triangle *EVS* is right-angled, with equal sides *EV* and *VS* adjacent to the right angle of triangle *EVS*. Using Pythagoras' theorem, it follows that the ratio of the size of the orbit of Venus to that of the orbit of the Earth, *SV* to *SE*, is 1 to $\sqrt{2}$: that is, 0.71 to 1.

This last calculation is simplified by assuming that the orbit of Venus is a perfect circle centered on the Sun. The deviations from this simplification complicate the determination only slightly.[24] A similar rearrangement gives the Copernican construction for Mercury and the determination of its orbital size.

The outer planets — Mars, Jupiter, and Saturn — required slightly different rearrangements. Their epicycles were not the representations of their true motions, merely the superposition of the Earth's motion onto their true motions. A similar analysis within the circles of the Copernican rearrangement gives the ratios of the sizes of these outer planetary orbits to that of the Earth. The analysis is a little more complicated. A greatly oversimplified version conveys the basic geometry of the analysis. Contrary to the reality, we assume that an outer planet is not moving. Then we can determine the ratio

24 For details, see Van Helden (1985, 43–44).

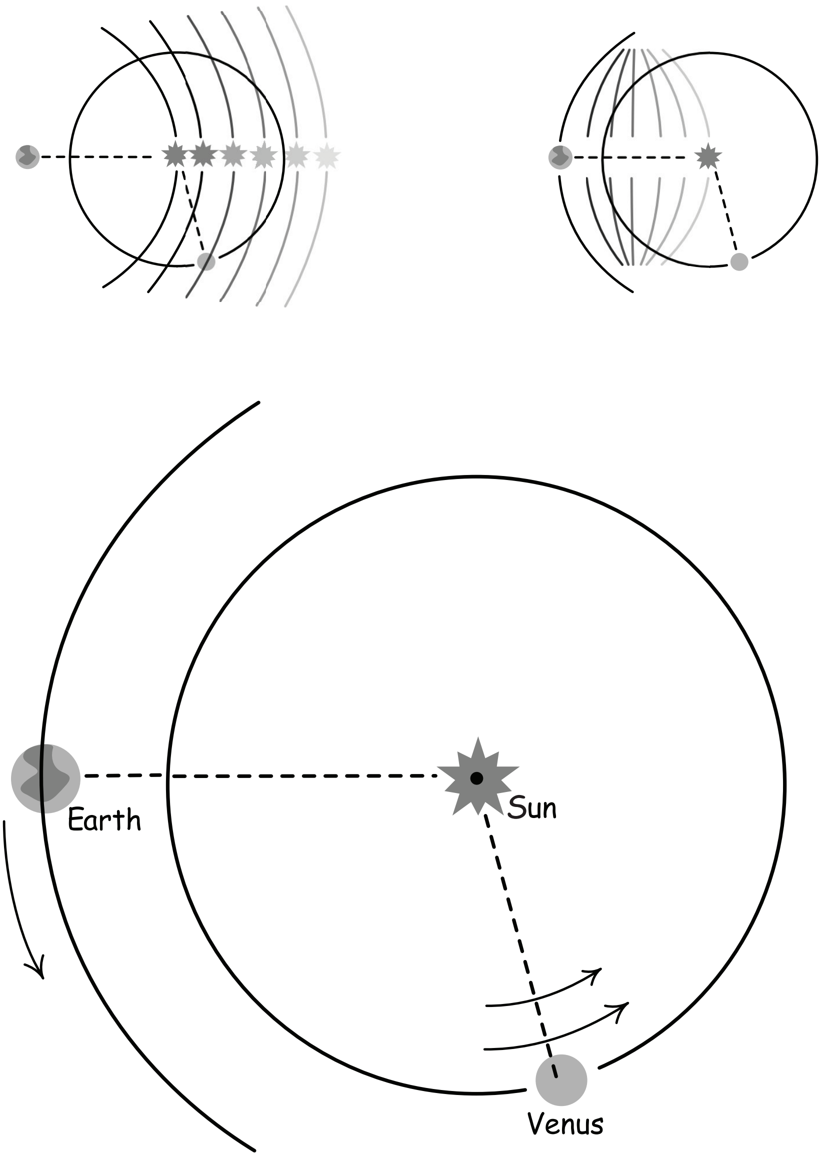

Figure 12.10. Venus in the Copernican system

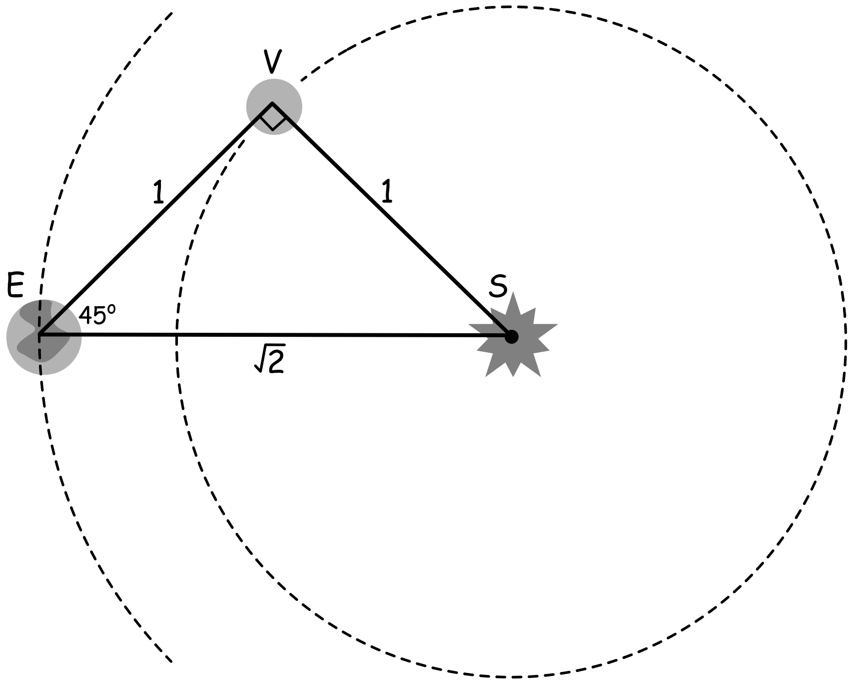

Figure 12.11. Fixing the size of the orbit of Venus

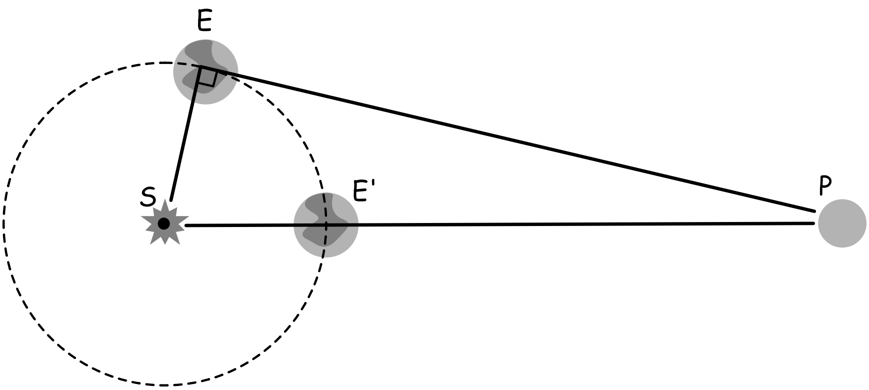

Figure 12.12. Distance to an outer planet

of the sizes of the orbits by checking how far the Earth progresses in its orbit between two orientations. First, the distant planet *P* is in direct opposition to the Sun *S*, indicated by the Earth at *E*' in Figure 12.12; second, the distant planet *P* is at quadrature — that is, at a right angle to the Earth-Sun distance — indicated by the Earth at *E* in Figure 12.12.

The angle *ESP* is known from how far the Earth has moved in its orbit. Observing the change in which stars are directly overhead at midnight would give the angle directly. Simple trigonometry on the right-angle triangle *SEP* tells us that the ratio of sizes *SP/SE* is 1/cos(*ESP*). This method is inapplicable in practice since the planet *P* will move during the time that the Earth progresses from *E*' to *E*. In the case of slow-moving Saturn, which has a period of 29.5 years, the movement will be slight. However, the analysis must correct for it. The correction is straightforward.[25]

11. Securing the Copernican Hypothesis

Copernican heliocentric astronomy and its later refinements proved to be key to the determination of planetary distances in the centuries that followed. It provided the ratios of the sizes of the orbits of the planets. All that astronomers needed was a single absolute measurement of one distance; then all of the rest could be recovered from the ratios. This was the procedure used after the seventeenth-century determination of the parallax of Mars and the eighteenth-century observations of the transits of Venus. This was the same strategy used by Ptolemy. His determination of the distance to the Moon triggered a cascade of computations that gave all of the distances. However, the difference was that independent evidence for Ptolemy's hypotheses never emerged. His inductive debt was never discharged. The Copernican hypothesis fared much better.

To begin, the Copernican system had an advantage over the Ptolemaic system in the practical challenges of securing evidential support. The Copernican system needed fewer independent hypotheses and thus fewer independent items of evidence. Ptolemy had to posit as an independent hypothesis that the centers of the epicycles of Mercury and Venus always aligned with the mean Sun, as shown in Figure 12.8. This alignment was automatic in the Copernican system since the center of the orbits of Mercury and Venus

25 For a simplified construction, see Crowe (2001, Chapter 6).

simply was the mean Sun. Similarly, Ptolemy had to posit that the epicycles of the outer planets — Mars, Jupiter, and Saturn — moved in perfect concert with the motion of the Sun, such that their retrograde motion coincided with their opposition to the Sun. Copernicus needed no such posits. These effects followed automatically from his recognition that these epicycles were merely the superposition of the Earth's annual motion on the true motions of the outer planets. Even just to recover an order for the planets in their distances from the Earth, Ptolemy had to posit additional hypotheses concerning their periods and motions. Copernicus needed no such additional posits. In his system, the relative distances of the planets from the Sun could be recovered from careful measurements of planetary positions.

As time passed, further evidence emerged. Galileo used his telescope to observe Venus in 1610, and he reported his results in his *Letters on Sunspots* of 1613. He saw Venus exhibiting a variety of Moon-like phases that could only be if its motion took it both closer to the Earth than the Sun and farther from the Earth than the Sun. This contradicted Ptolemy's system in which Venus is always closer to the Earth than the Sun but fit the Copernican hypothesis that Venus orbits the Sun.

It was Isaac Newton who made the decisive advance that fully discharged whatever residual inductive debt heliocentric astronomy might have carried. His *Principia* of 1687 provided a complete mechanics for the motions of the bodies in heliocentric astronomy. At the same time, celestial mechanics was combined with terrestrial mechanics in a single unified system. Any challenge to heliocentric cosmology would end up eventually having to challenge the entirety of this new physics.

12. Crossing of Relations of Support

The most useful relationship concerning the ratios of sizes of planetary orbits in the new astronomy is Kepler's so-called third law.[26] It asserts in its modern form that the square of the periods of a planet's orbit T^2 is proportional to the cube of the semi-major axis of its elliptical orbit R^3. Since the periods of two planets are accessible to measurement, the relationship provides a rapid determination of the ratios of their distances from the Sun. The relationship

26 Called thus, for example, by Maxwell (1894, 113).

between this law and Newton's mechanics provides a striking illustration of how relations of inductive support can cross over one another.

The distance-period relationship for the planets was first reported by Kepler for the mean distance from the Sun, among the many harmonies of his *Harmonices mundi* of 1619. In Book III of his *Principia*, Newton ([1726] 1962, 401–05) enumerated the phenomena from which his system of the world would be inferred. Phenomenon IV was his relation for the planets, asserted in terms of the mean distances. Phenomena I and II asserted the same relation for the moons of Jupiter and Saturn. Within Newton's mechanics, this relation could be translated almost immediately into a result central to his system: the acceleration due to the gravitational attraction of a body such as the Sun diminishes with the inverse square of distance. We can see how rapidly the result follows if we take the simple case of a planet or moon in a perfectly circular orbit of radius R with period T. It follows that the speed of the object is $V = 2\pi R/T$. Newton's mechanics sets the centrally directed acceleration A of such a motion equal to V/R^2. We can now combine these relations as

$$A = \frac{V^2}{R} = \frac{(2\pi)^2 R^2}{T^2} \cdot \frac{1}{R} = \frac{(2\pi)^2 R^3}{T^2} \cdot \frac{1}{R^2} = \text{constant} \cdot \frac{1}{R^2}$$

where Kepler's third law allows us to set R^3/T^2 to a constant.

Here we have the first relation of support:

from Kepler's third law *to* Newton's inverse square law of gravity.

It is possible to run the inferences in the above equalities in reverse and thereby infer Kepler's third law from the inverse square law:

$$A = \text{constant} \frac{1}{R^2} = \frac{V^2}{R} = \frac{(2\pi)^2 R^2}{T^2} \cdot \frac{1}{R} = \frac{(2\pi)^2 R^3}{T^2} \cdot \frac{1}{R^2}$$

We read from these equalities that R^3/T^2 must be a constant if we first assume the inverse square law. Thus, it is possible to have a relation of support that proceeds in the other direction:

from Newton's inverse square law of gravity *to* Kepler's third law.

Since the relation is a deduction, given the requisite background assumptions of Newton's mechanics, it is especially strong.

This second inference is commonly given in mechanics texts. Is it merely a formal derivation purely of mathematical interest? Or should we also conceive of it as a relation of evidential support proceeding in a direction opposite to that of Newton's original relation? That we can and should so conceive of it follows from a complication revealed by more careful analysis. The analysis above requires that the mass *S* of the central body, such as the Sun, should be considerably greater than the mass *P* of the orbiting body, such as a planet. When this assumption is relaxed, Maxwell (1894, 113–15) gives the correction that must be applied to the original form of Kepler's third law:

$$R^3 = \text{constant } (S+P)\, T^2$$

Deviations from the original law are small according to this formula as long as *P* is much smaller than *S*. However, for cases in which *P* becomes large in relation to *S*, the orbital periods will become smaller than predicted by the original relation from the orbital sizes. Maxwell proceeded to show that such deviations have been measured for the more massive planets Jupiter, Saturn, and Uranus.

Thus, Newton's mechanics does not merely recover Kepler's third law. Rather, it tells us the circumstances within which the law holds and gives a more general law that will hold when we deviate from those circumstances. In doing this, Newton's mechanics provides evidential support for Kepler's third law.

13. Conclusion

The determination of distances in our planetary system illustrates how hypotheses are used to extend the otherwise limited inductive reach of evidence. This is a procedure used widely in science. What makes the present case study revealing is that the investigations extended over millennia. That means that its stages are readily dissected. We can see in this slow development that evidence unaided by hypotheses was limited in its reach. Direct measurements of distances to celestial bodies by triangulation returned very little in spite of the most energetic and ingenious efforts. This reach was decisively furthered by various systems of hypotheses: harmonic, Ptolemaic, and Copernican. That each of the three considered here yielded different results underscores the provisional nature of the results. They are given a secure inductive foundation only when independent evidence is found for the hypotheses used and the inductive debt taken on in assuming them is discharged.

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