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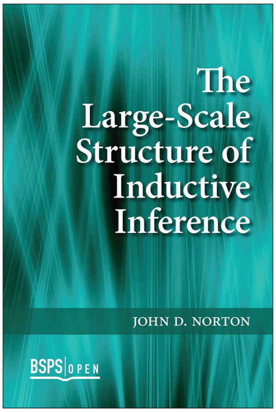

THE LARGE-SCALE STRUCTURE OF INDUCTIVE INFERENCE

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Large-Scale Structure: Four Claims

1. Introduction

In the previous chapter, I recounted how the material theory of induction treats relations of inductive support individually. That is, to what extent does this specific item of evidence support that proposition? If we think of inductive inference formally, this purely local examination might be sufficient. All that we need for a valid inference, according to a formal theory, is that the evidence and the supported proposition fit appropriately into the empty slots of some licit schema. This local appraisal is incomplete, however, when inductive inference is understood materially. In this approach, there is no fixed repertoire of warranted schemas applicable in all domains. In their place, (true) background facts in each domain warrant the inductive inferences supported in that domain. It follows that the affirmation that some inductive inference is licit requires a further affirmation of the truth of the background fact or facts that warrant the inference. These last facts themselves are contingent and, in the fullest account, must also be secured inductively with appropriate evidence.

Thus, when understood materially, the cogency of inductive inferences and relations of inductive support cannot be appraised fully in isolation. They must be appraised within the context of a larger ecology of relations of inductive support. In this book, I investigate how that larger ecology is configured. In this chapter, I lay the foundation of the material analysis of this large-scale structure. It consists of the following four claims, which I will introduce and defend in this chapter.

1. Relations of inductive support have a nonhierarchical structure.

2. Hypotheses, initially without known support, are used to erect nonhierarchical structures.
3. Locally deductive relations of support can be combined to produce an inductive totality.
4. There are self-supporting inductive structures.

In my defense of these four claims, I will employ extended examples drawn from the history of science. Providing a sufficiently detailed account of these examples within the confines of this chapter is impractical. My approach is to give these accounts in later chapters in Part II, with chapters devoted to each of the case studies. I will recall their results in this chapter briefly only insofar as they are needed.

In Section 2 of this chapter, I argue for the first and most important of the foundational claims listed above, the nonhierarchical structure of relations of inductive support. I address a supposition that relations of inductive support in science or in individual sciences are unidirectional, always proceeding from the less general to the more general. Under this supposition, these relations of support are akin to the relations of support among the successive courses of stones in a tower. Each course is supported only by those beneath it. In its place is a conception of greatly tangled relations of support that cross over one another, failing to respect any orderly hierarchy. They are akin to the relations of support in an arch or vaulted ceiling. Each stone is supported by those beneath it and many others above it and elsewhere distributed over the whole structure. That relations of inductive support form such a massively entangled system is the most prominent feature of the large-scale structure of relations of inductive support according to the material theory. Many further features will depend on it.

In Section 3, I ask how these entangled structures can be discovered. A central result of the material theory is that we first need to know something before we can infer inductively. Otherwise, we have no secure warranting facts for inductive inferences. If initially we know nothing in some domain, then how can we ever learn inductively generalities of infinite scope in the domain? An examination of episodes of scientific discovery gives the answer of the second claim: we proceed by hypothesis. That is, we introduce as hypotheses the facts that would be needed to warrant suitable inductive inferences, and then we make the inferences. In proceeding this way, however, we take on the obligation eventually to return to the hypotheses and provide independent support for them. Only then are our inductive inferences properly secured. The arches or vaulted ceilings of the analogy cannot be constructed simply by piling one stone upon another. To build them, we prop up some stones provisionally by scaffolding and complete the construction. Only then can the scaffolding be removed. The result is a structure each of whose stones, examined individually, is properly supported by masonry. This use of hypotheses is distinct from their use in hypothetico-deductive confirmation. There they are introduced in order to be confirmed themselves. Here they are introduced to mediate in the confirmation of other propositions.

In Section 4, I analyze the intriguing possibility asserted in the third claim found repeatedly realized in cases of inductive support in science. In many cases, the component relations among propositions are individually deductive, even though their combined import is inductive. In this section, I will recall some examples that show how combinations of deductive relations among propositions can, overall, have inductive import.

As a prelude to discussion of the fourth claim, in Section 5 I characterize a mature science as inductively rigid. That means that each proposition of the mature science enjoys strong inductive support from the evidence and that the evidence admits no alternatives. Such a system is intolerant of challenges and generally repels them. If they are successful, then they have a destructive, revolutionary effect. A cascade of strong relations of evidential support propagating through the science will have to be undone.

In Section 6, I develop the fourth claim of the possibility of a self-supporting inductive structure. It is a closed structure in which each proposition is well supported inductively by evidence in the structure through warranting propositions also in the structure. A mature science forms such a structure if we expand its compass to include all of the propositions warranting its inductive inferences, and the evidence and warrants for them, and so on to closure. To see the self-supporting inductive structure, pick any proposition in the science. All of the evidence and warranting propositions needed for its inductive support will be in the structure. That is just the condition that it is inductively self-supporting.

In Section 7, I consider the possibility of nonempirical conditions that might be a necessary supplement to a complete account of the large-scale structure of inductive inference. One might look to a priori principles such as a principle of causality or to the remarkable success of mathematics in formulating physical theories. Such added components, it is argued, fail insofar as they have no empirical foundation; if they do have an empirical foundation, then they lie within the material theory.

In Section 8, I provide a brief preview of what is to come.

2. Nonhierarchical Relations of Inductive Support

Relations of inductive support have a nonhierarchical structure.

2.1. The Hierarchical Conception: The Tower

The original and simplest notion of inductive inference is the notion of generalization from instances. It is codified in the schema of enumerative induction and employed in embellished form by time-honored procedures such as Bacon's tables and Mill's methods. It promotes an oversimplified image of science as an accumulation of generalizations of successively broader scope.

Here is how it looks. In biology, we might start with the particular observations of the flora and fauna of Europe and form generalizations from them. We might then expand our inductive base with particular observations of the flora and fauna of the Middle East, Africa, and Asia. Generalizations concerning them are combined with the earlier generalizations concerning European flora and fauna. We then expand our inductive base even further by introducing knowledge of biological species in the Americas and then the Antipodes. New generalizations concerning them are combined with those achieved earlier to yield generalizations of still greater scope.

We can find similar structures in other sciences. In physical astronomy, we note with Newton that all bodies on Earth gravitate and that all celestial bodies gravitate. We combine the two generalizations to arrive at the greater generalization that all matter gravitates. We note that our Moon and the moons visible to us are nearly spherical, so we infer that all moons are nearly spherical. We infer the same for planets and then eventually for suns and stars.

The result is a stratification of the propositions of a science according to their generality. At the bottom are the least general, the particular facts, commonly conceived as facts of experience or possible experience. As we ascend the hierarchy, we pass to generalizations from them, and then generalizations from them, and so on. The generalizations of the higher layers are supported inductively by those of the lower layers. We descend in the hierarchy by making deductive inferences. They take us from generalizations higher in the hierarchy to those lower in it.

This hierarchy is analogous to the structural support relations among stones in a tower, shown in Figure 2.1. The first course of stones sits on firm ground. It supports the next course of stones, which supports the one above it, and so on to the top of the tower. The firm ground is analogous to experience. It supports the simplest propositions of experience, commonly conceived as propositions about particulars. Each course of stones structurally supports those above it, just as generalizations lower in the hierarchy inductively support those higher up in it.

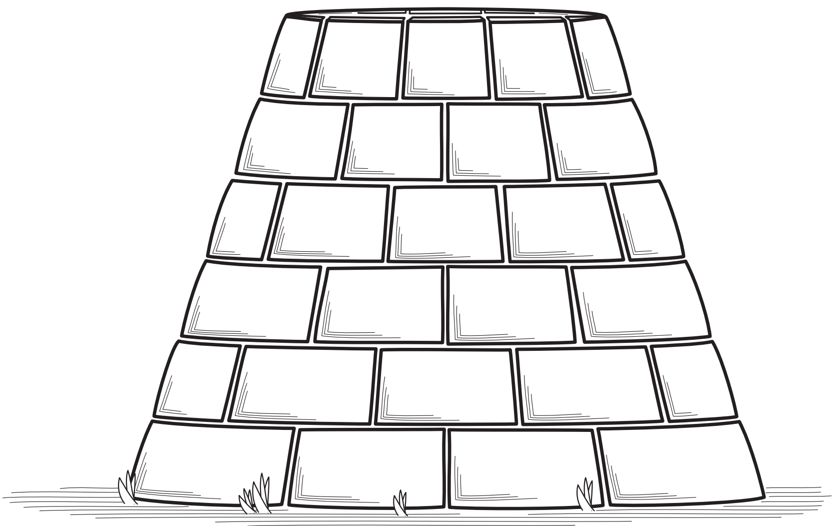

Figure 2.1. A tower

Although a hierarchical structure of this sort sometimes appears in science, overall it is a poor representation of the organization of propositions in science and the inductive relations among them. It fails for at least two reasons. First (to be developed in Section 2.2), contrary to the tacit supposition, relations of inductive support do not respect the hierarchy of generality. Second (to be developed in Section 2.3), the propositions of science are sufficiently varied in content that their strict partitioning and ordering by generality are unsustainable.

2.2. Relations of Inductive Support Do not Respect the Hierarchy

The hierarchical presumption is that relations of inductive support are unidirectional: they proceed from the less general to the more general. A closer examination of the relations of inductive support within a science shows that this unidirectionality is not respected. Relations of support typically cross over one another. Speaking now only loosely of comparisons of greater and lesser generality, propositions at one level of generality can be supported by a combination of propositions of lesser, equal, or greater generality. The relations commonly are so tangled that no simple ordering of their direction by generality among the propositions of a science is possible.

We shall see more examples below of this lack of respect. It is worth pausing here to visit an especially striking example. It is provided in Chapter 7, "The Recession of the Nebulae." In 1929, Edwin Hubble announced the result that would become the observational foundation of modern cosmological models. Nebulae[1] recede from us with velocities linearly proportional to their distances. Superficially, his analysis looks like the simplest of generalizations. Hubble reported as data the velocities of recession of individual nebulae, as inferred from red shifts in their light, and the distances to these nebulae. This is the level of lesser generality in the hierarchy. He then formed a generalization about all nebulae: their velocities of recession vary linearly with their distances. This generalization resides at a higher level of greater generality in the hierarchy.

Hubble's generalization, it seems, proceeded as we might naively expect, unidirectionally up the hierarchy. As the later chapter shows, his actual inferences were far more complicated and quite unconstrained by this hierarchy. Most troublesome of several problems was that Hubble lacked almost half of the requisite independent distance measurements. His data set reported velocities for forty-six nebulae but included independently derived distance estimates for only twenty-four of them. Hubble was determined, however, to include all forty-six nebulae in his analysis and employed inductive stratagems of some ingenuity and complexity to proceed. In one prominent case, he *assumed* the generality of a linear relationship between the velocities and distances and used it to infer the unknown distances. This inference mixed

1 Hubble's "extragalactic nebulae" or just "nebulae" are, of course, now called "galaxies."

elements from the less general and more general levels to infer propositions in the less general level. Hubble could then test that the inference was successful by using the inferred distances to recover the absolute magnitudes of the nebulae concerned. He checked that these inferred absolute magnitudes conformed to other nebulae of independently known absolute magnitudes.

2.3. The Hierarchy of Generalizations Is Unsustainable

The second false presumption in the hierarchical conception is that it is possible everywhere to partition and order the propositions of a science by generality. Although something like this might be possible in simpler contexts, the presumed partitioning and ordering become impossible to maintain as the propositions of science become more abstract and remote from the specific propositions of observation and experiment. No simple sequence of successive generalizations takes us from the chemical reactions observed in a laboratory to the bonding theory of the complex molecules of organic chemistry, or from the observed emission spectra of gases to the quantum mechanics of the electrons of atoms, or from the motions of the planets to the curved spacetime geometry of general relativity. The inductive pathways from simpler observations and experimental results to the completed theories are sufficiently convoluted that there is no evident basis for comparisons of generality among the intermediate propositions.

For example, ordinary Newtonian mechanics in its various parts treats the distribution of stresses in bodies, the motions of terrestrial projectiles, the flows of fluids, the motions of planets, and much more. How do we rank their many propositions according to their generalities? Is the theory of the distribution of the many stress forces in a complicated architectural structure more general than the analysis of the few gravitational forces acting in a simple problem in orbital mechanics? Or is the latter more general since it treats not just forces but also the motions that they produce? In chemistry, the energy states of a single hydrogen atom are treated by quantum mechanics. Prior to its quantum treatment, the chemistry of hydrogen is treated by a simple phenomenological theory telling us that gaseous hydrogen consists of molecules in which two hydrogen atoms bond. Is the phenomenological theory of the hydrogen molecule more general because it treats bonded hydrogen, whereas the quantum theory of individual atoms does not? Or is the quantum treatment of the hydrogen atom more general since it is part of the more advanced quantum treatment of chemical bonding in which the energy levels of the hydrogen atom play a central role? These questions, and many more like them across the sciences, admit no well-founded answers.

2.4. The Arch

There is no overall partitioning and ordering of the propositions of science by generality. Even when such local orderings appear, relations of inductive support do not respect them. Instead, relations of inductive support are distributed over the propositions of science in a massively entangled network. The simplest instances of this entangled network arise in a crossing over of relations of support whenever we have highly correlated properties. Then a proposition concerning one property can provide support for others at what we might loosely judge to be a comparable level of generality, and those others can provide support in reverse for the original proposition. These relations of support are warranted in turn by the more general proposition of the correlation itself.

For example, stars can vary in many properties, including their effective temperatures, masses, sizes, and elements. A class O star in the Harvard spectral classification system is rare, characterized by a very high effective temperature of the order of 30,000K or greater. Many other properties of stars are strongly correlated with this temperature. A class O star will also have a huge mass and a tremendous luminosity.

Exactly because all of these properties are highly correlated and otherwise unusual, finding one of them in some new star is strong evidence for each of the others. For example, finding that a newly observed star has a very high effective temperature greater than 30,000K is strong evidence that the star is massive. The converse also holds: finding that the star is massive is strong evidence that it has a very high effective temperature. This crossing over of evidential support can be continued for other pairings of properties of class O stars.

There is an architectural analogy to this pair of propositions, each of which provides an inductive warrant for the other. It replaces the analogy to the tower. It is an arch, shown in Figure 2.2. Each side of the arch rests on the firm ground of experience. However, none of the stones higher in the arch is merely supported by the stones beneath it. The stones are also supported by those still higher in the arch and ultimately by those of the other side. One side of the arch, if built without the other, would simple fall down. The two sides mutually support one another.

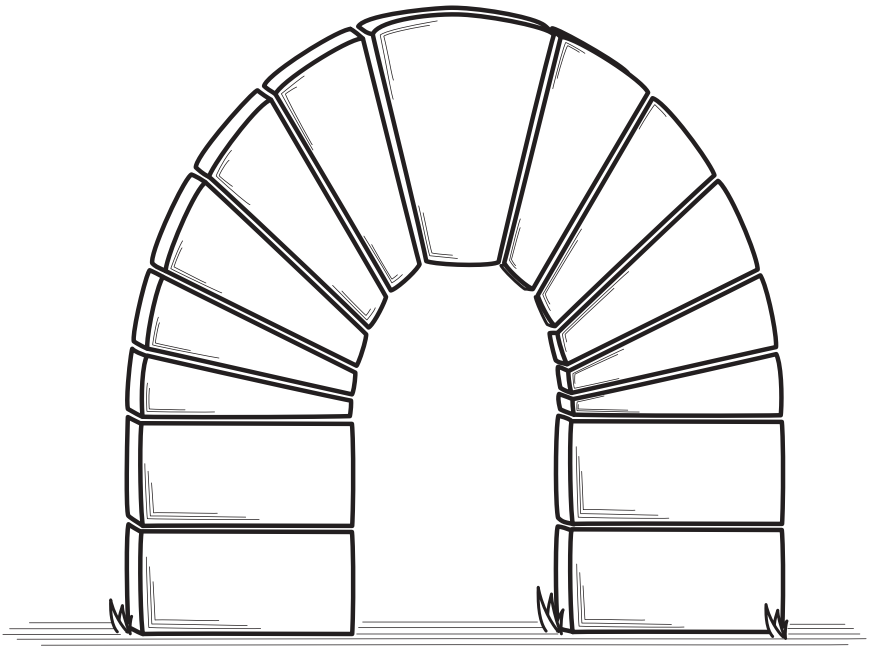

Figure 2.2. An arch

2.5. Arches Illustrated

In later chapters, I provide more examples of this arch-like crossing over of relations of inductive support. In Chapter 8, "Newton on Universal Gravitation," I describe two cases of pairs of propositions that mutually support each other. The first arises in his "Moon test," in which he argues for the identity of the force of gravity and the celestial force that holds the Moon in its orbit around the Earth. The evidence is the observed accelerations of the Moon toward the Earth and falling bodies at the surface of the Earth. Newton computes the acceleration that the celestial force would yield if it acted at the Earth's surface while strengthening according to an inverse square law. He finds that acceleration to match the observed acceleration of bodies falling at the Earth's surface.

Consider the proposition that the celestial force on the Moon strengthens according to an inverse square law with distance. In this first inference, it is used as an inferential warrant in arriving at the identity of the celestial and terrestrial forces. This usage can be reversed. The proposition of the identity of the celestial and terrestrial forces can also be used as a warrant. Then one can infer from the observed motions that the celestial-gravitational force acting on the Moon strengthens with distance according to an inverse square law. That is, the proposition of the identity of celestial and gravitational force and the proposition of the inverse square law mutually support one another.

In a second example in his account, Newton fits elliptical orbits to the observed positions of the planets. The inference from these positions to their specific elliptical orbits is warranted by the proposition that the planets are acted on by an inverse square law of gravity. Excluding perturbations, that law entails that planets move in conic sections: ellipses, hyperbolas, or parabolas. However, a second argument reverses the proposition that warrants the proposition supported. The key warranting fact is that the elliptical orbits are reentrant. Each planetary year a planet follows the same elliptical orbit. This reentrance, Newton shows, can arise only with an inverse square law of gravity. Taking them together, we find that the specific elliptical orbits of the planets support the inverse square law and that the inverse square law supports the specific elliptical orbits of the planets.

Radiocarbon dating of artifacts provides another illustration of this crossing over of relations of support. It is described in Chapter 10, "Mutually Supporting Evidence in Radiocarbon Dating." In the simplest description, there are two sorts of propositions concerning the dating of artifacts. The H propositions date them by the traditional methods of historical analysis and archaeology. The R propositions date them by estimating how long it took for their content of the radioactively unstable isotope of ^{14}C to decay to the measured levels. The R propositions depend on an accurate knowledge of the original content of ^{14}C captured in artifacts at their formation in different epochs. This knowledge is provided by H propositions: the dating of artifacts by traditional methods. Here H propositions provide evidential support for R propositions. However, the reverse can also happen. Are we sure that no error has crept into the historical methods used to arrive at a traditionally established dating? Then radiocarbon dating can reassure us or correct us. Now R propositions are providing evidential support for H propositions.

I provide more examples of mutually supporting pairs of hypotheses in other chapters. In Chapter 11, "The Determination of Atomic Weights," we see how Avogadro's hypothesis and the law of Dulong and Petit supported each other in chemical investigations of the early nineteenth century. The same relation of mutual support later arose among the chemists' version of Avogadro's hypothesis and the physicists' version of the hypothesis within the kinetic theory of gases. In Chapter 9, "Mutually Supporting Evidence in Atomic Spectra," we find the Ritz combination principle providing support for the quantum theory. Then later the quantum theory provides support for a corrected version of the Ritz combination principle.

2.6. The Vaulted Ceiling

The examples above of pairs of mutually supporting propositions are exceptional for their simplicity. It is far more common for these relations of mutual support to be embedded within a much larger network of inductive relations of support in a science. The Newtonian example is not of an *isolated* structure since the various hypotheses in it figure in relations of support for other propositions in science.[2] In general, relations of support cross over one another in many different ways and at many different levels. One then finds that even a small part of science can be part of a prodigious array of relations of support connecting it with neighboring sciences and then beyond them to the farthest reaches of science.

The analogy to a single arch does not capture this richness. An analogy to a dome is a little better. Stones in each part of the dome depend for their support on stones in many other parts. A still better analogy is to a massively complicated vaulted ceiling, as shown in Figure 2.3. It consists of many interconnected domes and arches. The integrity of the entire structure depends on the mutual support of all of its parts.

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2 For example, an inverse square law is presumed in the computations associated with Cavendish-type experiments that determine the magnitude of the gravitational constant G. The law is also used to infer that spherical planets act gravitationally, as if their masses were concentrated at their centers; to infer that certain comets move on hyperbolas; and to compute the behavior of terrestrial tides.

Figure 2.3. A vaulted ceiling in the Commons Room, Cathedral of Learning, University of Pittsburgh; image by John D. Norton

This interconnectedness of relations of inductive support provides mature science with its monolithic structure. One cannot reverse one part without destabilizing the remainder of the structure. A vivid example of an effort to reverse one part comes with the persistent creationist efforts to remove evolutionary theory from biology. The problem that creationists face is that evolutionary theory is inductively entangled with the other sciences. In their challenge to that theory, creationists find that they need to impugn the great age of the Earth in favor of a much younger Earth, whose age is determined from biblical scholarship. Hence, they must impugn modern uniformitarian geology. It is based on an old Earth whose major geological features were formed slowly over eons. They must impugn the radiological methods used to date both organic artifacts and rocks, which ultimately will lead to conflicts with radiochemistry. They must also dispute standard cosmology since it also calls for an ancient Earth. This forces them, then, to question observational and theoretical astronomy and the physics on which it depends.

The size of the network of support relations in mature sciences leads to a combinatorial explosion in the number of support relations that directly or indirectly bear on the propositions of the component sciences. This effect gives depth to the inductive security of each part. A fully worked-out example would help us to see this security more clearly. Unfortunately, displaying the complexity of such a network in all of its detail is a task too large for this chapter or this book. However, we can get a good sense of the density and richness of these structures by visiting just small pieces of them in the examples developed in the chapters that follow.

2.7. Vaulted Ceilings Illustrated

In Chapter 11, "The Determination of Atomic Weights," I recount the immense difficulties faced by chemists in the early nineteenth century in determining relative weights of atoms. The problem arose in Dalton's *New System of Chemical Philosophy* of 1808 and 1810. Dalton knew, for example, that 8g of oxygen combines with 1g of hydrogen to make water. To infer from this that the molecular formula of water is H_2O, he needed to know that an oxygen atom is sixteen times as massive as a hydrogen atom. He had no table of atomic weights to consult and no way to determine them, so he just assumed that the ratio was 8-1. The result was that he arrived, famously, at the molecular formula for water of HO. Dalton was trapped in a circularity: to know the correct molecular formulae, he needed to know the relative weights of atoms, but he could learn the relative weights of atoms only from the molecular formulae.

One might imagine that this circularity was easily broken. It was not. The task required the efforts of chemists over roughly half a century. Chapter 11 recounts Cannizzaro's celebrated solution circulated at the Karlsruhe conference of chemists in 1860. Cannizzaro relied on Avogadro's hypothesis, the law of Dulong and Petit, and an extensive set of measurements of the physical properties of a wide range of substances to determine their molecular formulae. The determinations were complicated, and I have done my best to present them in Chapter 11. For my purposes here, the key fact is that the molecular formulae were not just determined but also overdetermined. That means that some subset of them could be used to provide inductive support from some other part and vice versa.

For example, once Cannizzaro had determined that hydrogen and oxygen gases are diatomic, H_2 and O_2, his gas density data enabled him to fix the molecular formula for water as H_2O. Or he could start with this molecular formula for water and find that oxygen and hydrogen are diatomic. This is just a glimpse of a massive tangle of relations of inductive support in Cannizzaro's analysis. For example, that hydrogen gas is diatomic entered into similar overdetermined relations of support concerning compounds of the halogens: chlorine, bromine, and iodine.

In Chapter 9, "Mutually Supporting Evidence in Atomic Spectra," I provide another illustration of this sort of tangle of relations of inductive support. Energetically excited hydrogen gas emits light. It emits only specific frequencies of light whose measurement became an important project for spectroscopists in the late nineteenth century and early twentieth century. Those frequencies divided into well-structured sets of lines, found in different parts of the electromagnetic spectrum: the infrared, the visible, and the ultraviolet. These sets or "series" were named after the spectroscopists who measured them: the Lyman, Balmer, Paschen, Brackett, and Pfund series.

The series were connected by a simple arithmetic relationship first noted by Rydberg but exploited by Ritz in 1908 as his "principle of combination." The key fact was that the lines of all of the series could be generated by taking the arithmetic differences of a set of terms. For Ritz, this fact provided a useful heuristic. He could apply his combination principle to the lines of a known series and predict a new, hitherto unobserved, series. The approach proved to be successful, and immediately Ritz could report a new line conforming with his prediction.

For my purposes, what is important is that the full set of lines in all of these series is overdetermined once one adopts Ritz's principle. That means that one can take the lines of one series and infer from them to the existence of another series. What results is a tangle of relations of inductive support. This structure, fortunately, is much easier to comprehend, as Chapter 9 shows, since it is recoverable by simple arithmetic additions and subtractions.

2.8. The Firm Ground of Experience

In the arch and vaulted ceiling analogies, the ground that supports the masonry corresponds to the empirical basis of the science. This basis does not depend on any simple-minded or strict distinction between observational and theoretical propositions, for I follow the now common view that a clear distinction between them cannot be made. Rather, I mean by it what is commonly taken in a present science as its supporting empirical facts. They can be far removed from direct human observations.

For example, one of the most stable and most important observational facts supporting modern cosmology is that space is filled with a 2.7K background of thermal radiation. This simple-sounding fact was secured over decades only after extraordinary efforts, some of which are recounted in Chapter 9, "Inference to the Best Explanation: Examples," of *The Material Theory of Induction* (Norton 2021). Among the difficulties faced, to establish a thermal character in a radiation field, one must have measurements made at many different frequencies. Only then can the energy distribution characteristic of thermal radiation be established.

A related observational fact of modern cosmology is that galaxies are observed to recede from us with a velocity that increases linearly with distance. Although the observation is now routinely reported without much hesitation in modern treatments, it was subject to a searching critique in the later twentieth century by Halton Arp. He argued that the red shift in light from the galaxies could not be interpreted as resulting from a velocity of recession since objects with very different red shifts appeared to be connected spatially. An extensive debate was needed to refute his hesitations (for details, see Norton 2023).

The analysis of just what might be meant by the empirical facts of a science is a project that goes beyond my concerns here. My view is that Nora Boyd's (2018a, 2018b) analysis provides the best modern treatment. Boyd allows that all such empirical facts are entangled with theory. However, she argues, these facts can still be used to decide among competing theories through a process of winding back to the provenance of the facts. When we seek to use some empirical fact to decide between two theories, we wind back through the various stages of the formation of the fact. If sufficient data have been preserved, then eventually we come to a point at which enough of the theoretical encumbrance has been removed for the fact to provide a neutral basis of comparison for the two theories.

3. The Role of Hypotheses in the Discovery of Inductive Relations of Support

Hypotheses, initially without known support, are used to erect nonhierarchical structures.

3.1. The Discovery Problem

The discussion in the previous section concerns relations of inductive support, independent of human knowledge of them. A further question of great importance is how we can learn these relations. Only then do they assist us in our inductive exploration of the world. If the totality of facts connected by relations of inductive support were delivered to us as a completed whole, then it would be a straightforward matter to check that all of the requisite relations of inductive support obtain. This is a science fiction scenario. It is what would happen were we to stumble onto a copy of the fictional *Encyclopedia Galactica* of some advanced alien civilization. In it, entire sciences hitherto unknown to us would be delivered to us in their totality.

In real life, our explorations proceed more haltingly. The guiding rule of the material theory of induction is that "you must already know something to be able to infer inductively." We cannot know that some inductive inference is licit unless we are assured of the truth of the warranting fact. Yet, if we are in the early stages of investigation in some new field, then commonly we know rather little, and it is likely too little to proceed with assured inductive inferences of any great reach.

This is a problem faced by all new sciences. The strategy used almost universally is to proceed provisionally. We might not know which are the general facts of some domain, but sometimes we can determine which propositions are plausible candidates for the facts that would warrant the inductive inferences sought. To use a familiar term, these plausible propositions are "hypotheses." We can then proceed provisionally under the supposition that our hypothesis is a fact and infer to the propositions that it would warrant were it a fact. The key element is that the supposition is provisional. Conclusions drawn or inductively supported using the hypothesis themselves have only provisional status. They remain so until we find inductive support for the warranting hypothesis. We have incurred an inductive debt in proceeding to the conclusions, and they are properly secured only when that inductive debt is discharged by finding support for the warranting hypothesis.

Hypotheses have a natural analogue in the procedures for building arches, domes, and vaulted ceilings. A masonry arch, dome, or vaulted ceiling cannot be built simply by piling stones one upon another. As soon as a few stones have been placed, the highest ones would be without adequate support and would fall. The standard procedure is to use scaffolding, known technically as "centering." As shown in Figure 2.4, traditionally it consists of a wooden framework. The stones are set on top of the framework. Prior to the completion of an arch, these stones are not properly supported by its other stones. Their support is only provisional since the wooden centering will be removed eventually. Here they are analogous to hypotheses whose support is also only provisional. When all of the stones of the arch have been placed, the centering can be removed. Now the remaining stones of the arch fully support each other. This final stage of construction is analogous to discharging the evidential debt taken by introducing the hypothesis. As the full investigation is completed, further inductive support, anchored eventually in experience, is provided for it.

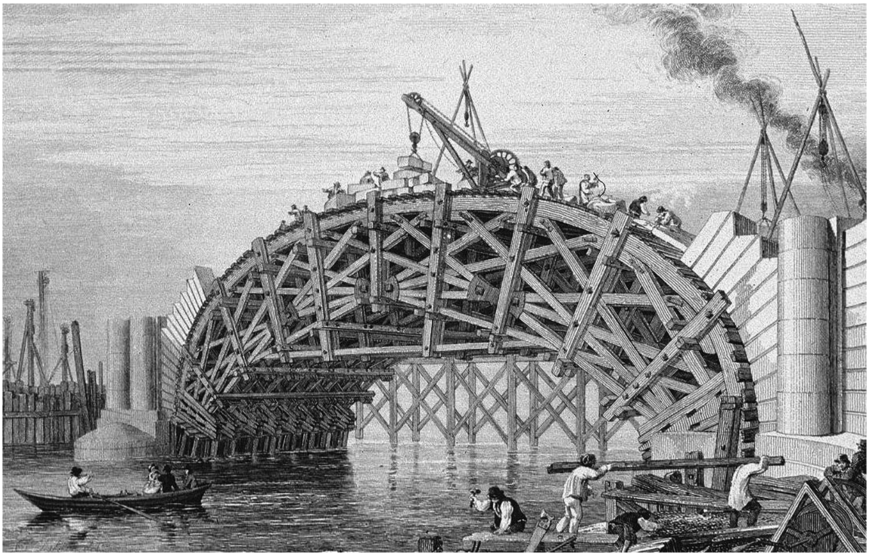

Figure 2.4. Wooden centering used in the construction of the Waterloo Bridge

3.2. Hypotheses Illustrated

The chapters that follow provide illustrations of this use of hypotheses. In several of them, the use of hypotheses is invited by a specific problem. Scientists find themselves trapped in an evidential circle. Commonly, there are two related quantities to be determined. To find the first, the scientists need to know the second, but initially it seems that they cannot know the second unless they already know the first. They are trapped. A suitably chosen hypothesis is used routinely to break the circle.

Chapter 12, "The Use of Hypotheses in Determining Distances in Our Planetary System," is an extended study of this use of hypotheses. Consider the earliest efforts to determine distances to celestial bodies. The Moon subtends an angle of about half a degree in our visual field. If we knew the diameter of the Moon, then simple geometry would let us compute the distance to the Moon. However, we do not know its diameter precisely because we do not know how far it is from us. Determining its distance and diameter forms the troublesome evidential circle. The Sun also subtends an angle of about half a degree in our visual field. Determining its distance from us is blocked by the same evidential circle. Determining distances to the planets is even harder since naked eye astronomy cannot resolve their disks. They are just points of light in the sky.

Chapter 12 recounts how ancient and later astronomers sought to break out of this evidential circle by ingenious geometrical triangulations or, as it is known in the astronomical context, measuring parallax. These efforts met with limited success. Ancient astronomers were unable to measure the tiny parallactic angles accurately enough. In the seventeenth century, using telescopic aids, a fairly good parallactic measurement of the distance to Mars was achieved. However, even with telescopic aids, *direct* parallactic measurements of the key Earth-Sun distance were not achieved as late as the nineteenth century.

From the outset, to fill the gaps, hypotheses were called into service. They were not used to fix the distances directly, only to provide hypothetical estimates of the ratios of the distances. Then all that was needed was a single distance determination, such as the distance to the Moon or to Mars, and the remaining distances could be computed from the ratios. What makes this case study revealing is that, in addition to a success story, it recounts failures. They arose when independent evidential support could not be secured for the hypotheses, and eventually they were rejected. The chapter recounts three attempts.

The earliest were Pythagorean/Platonic proposals that recovered the ratios from musical harmonies and simple arithmetic relations. A later proposal was incorporated into Ptolemy's geocentric cosmology. Ptolemy proposed a plausible distance ordering for the celestial bodies. He recovered the ratios of their distances from the further hypothesis that their orbits are packed together as closely as the geometry of the compounded circles of his system allowed if intersections of the circles are precluded. Neither Pythagorean nor Ptolemaic proposals were able to secure independent evidence. Their inductive debt was not discharged, and they were abandoned.

They were replaced by Copernicus' , heliocentric hypothesis. Through it, the ratios of the planetary orbital distances were readily recoverable from terrestrial measurements. Unlike the earlier systems, the Copernican hypotheses gained evidential support from both within and without. Most important was its conformity with Newton's mechanics. Newton had used the more fully developed heliocentric astronomy of his time as an essential premise of his argument for universal gravitation. In another example of the crossing over of relations of inductive support, the direction of inductive support was reversed. Newton's mechanics soon became strong evidence for the details of Copernican astronomy.[3]

The dependence of solar system distance measurements on the heliocentric theory persisted. The most accurate estimates of the key Earth-Sun distance in the eighteenth and nineteenth centuries came from careful measurements of the transits of Venus across the face of the Sun. The Earth-Sun distance could then be recovered from them by geometric triangulations. These calculations still relied on the heliocentric theory's determination of the ratios of the orbits of the Earth and Venus.

Further illustrations of the use of hypotheses to break evidential impasses have already appeared earlier in this chapter. We saw how Dalton was trapped in an evidential circle concerning atomic weights and molecular formulae. He sought to break the circularity by hypothesizing that the correct molecular formulae used the simplest ratios available. The hypothesis failed to secure independent support and was abandoned. The circularity was broken later through two hypotheses: Avogadro's hypothesis and the law of Dulong and Petit. The evidential debt incurred in supposing them was discharged eventually through the mutual support of these two hypotheses and the support provided for them from the emergence of the statistical mechanical treatment of gases in physics.

We also saw that Hubble was stymied in his efforts to use the data from all forty-six nebulae for which he had measurements by a lack of independent distance measurements for twenty-two of them. Chapter 7, "The Recession of

3 The inversion in this relationship is seen most clearly in the ability of the Newtonian system to provide corrections to the heliocentric astronomy of Newton's time. The planets orbit not in ellipses but in precessing ellipses. What came to be known as Kepler's third harmonic law was corrected to accommodate the finite mass of the Sun. The importance of successive approximations in Newton's and later work has been explored by Smith (2002, 2014).

the Nebulae," recounts how Hubble was still able to incorporate these twenty-two nebulae into his analysis by means of hypotheses that gave him indirect indications of their distances. At various stages of his analysis, he hypothesized that the linear relationship among the other twenty-four nebulae also held for these twenty-two, that the absolute magnitude of the brightest star in each nebula is the same, and that the absolute magnitudes of nebulae in a cluster are confined to a small range common to all nebulae.

In the early-twentieth-century analysis of atomic spectra, we saw how the discovery of new series was advanced by the Ritz combination principle. It was introduced as a hypothesis. It gained the requisite independent evidential support with the emergence of modern quantum theory, in which it was recovered as a consequence of Bohr's atomic theory.

These last illustrations have been mostly of successes secured at least eventually. This happy outcome is not assured. A prominent example of a failure is provided by the steady state cosmology of the mid-twentieth century. It was based on the hypothesis of the "perfect cosmological principle," first advanced by Bondi and Gold (1948). According to it, the universe is homogeneous on the large scale, not just spatially but also over time. The way in which we see the universe now, on the large scale, is the way in which it has always been and will always be. A definite cosmology now follows. Its most striking feature is the continuous creation of matter. Unless matter is continually created throughout space, expansion of the galaxies would lead to a dilution of its average matter density and violate the perfect cosmological principle. The steady state cosmologists took on a massive evidential debt in hypothesizing the perfect cosmological principle. They were never able to establish independent evidence for the hypothesis, and they were never able to repay the debt. Most notable was the failure of the steady state theorists to accommodate Penzias and Wilson's discovery in 1965 of the cosmic background radiation, and the competing "big bang" or "primeval fireball" hypothesis eventually proved to accommodate it handily.[4]

3.3. This is *not* Hypothetico-Deductive Confirmation

This use of hypotheses might appear to be similar to the hypothetico-deductive approach to confirmation. These are accounts of confirmation based on the principle that a hypothesis is inductively supported when it

4 For a brief account of this last competition, see Norton (2021, Chapter 9).

successfully entails true evidence deductively.[5] The essential difference lies in the goal of introducing the hypotheses in an evidential analysis. In hypothetico-deductive confirmation, hypotheses are introduced so that the evidence can confirm them according to the hypothetico-deductive principle. In the applications within the material theory, hypotheses are introduced to mediate in the confirmation of *other* propositions. The confirmation of the hypothesis is a task reserved for later investigations. The hypothesis is expected to be confirmed not hypothetico-deductively but by other inductive inferences with their own material warranting facts.

4. Deductive Inferences in Inductive Structures

Locally deductive relations of support can be combined to produce an inductive totality.

4.1. Inferences that Are or Are Nearly Deductive

There is a striking feature of many of the inferences in this text and in my earlier text, *The Material Theory of Induction* (Norton 2021). Although the inferences contribute to relations of inductive support, many of them are close to being deductive inferences or might actually be deductive inferences. That is, when combined with the warranting fact, the inference *from* the evidence *to* the conclusion to be supported is often deductive. The direction of the inference here is important. It is not merely the deductive inferences of hypothetico-deductive support. In the latter, the deduction passes from the hypothesis or theory to the evidence. That direction has now been reversed.

Here are some examples. Chapter 1 of *The Material Theory of Induction* (Norton 2021) recalled Curie's inference from the crystallographic properties of the few samples of radium chloride at her disposal. Curie inferred to the generality of these crystallographic properties. I identified the warrant for her inference as

> (Weakened Haüy's Principle) *Generally*, each crystalline substance has a single characteristic crystallographic form.

5 For an elaboration of this principle and the extensive problems associated with it, see Norton (2005).

When this weakened principle is used to warrant Curie's inference, it is the qualification "generally" that makes the inference inductive. It accommodates the possibility of polymorphism, that one crystalline substance might manifest in more than one crystallographic form. The inductive risk taken by Curie is small, especially if we assume that her generalization was tacitly limited to crystals of radium chloride prepared under conditions comparable to those in her laboratory.[6] If we drop this qualification and revert to Haüy's original conception, the warranting fact would be

> (Haüy's Principle) Each crystalline substance has a single characteristic crystallographic form.

Under this warrant, Curie's inference would be a deduction.

Chapter 2 of *The Material Theory of Induction* (Norton 2021) recounted Galileo's inference concerning his law of falling bodies. Galileo had found that, in equal time intervals, a body in free fall successively covers distances in the ratios of 1-3 to 5-7. He generalized this sequence of ratios to the sequence of odd numbers. In this inference, I argued that Galileo had used the warranting fact that the ratios of 1-3 to 5-7 were present no matter the time interval used in the measurement. It then followed, deductively, that the only possible general law was of the sequence of odd numbers. Indeed, the deductive inference needs as a premise only the ratio of 1-3 and its invariance under a change of the unit of time.

There are, it turns out, other well-recognized, historically important examples in which the inference from evidence to our theories is deductive. These cases have been codified as "demonstrative inductions." Their inferences are demonstrative in the sense that they are deductions. However, they are called "inductions" to reflect an older usage of the term as referring to inferences from particularities to generalities. My contribution to this literature in Norton (1993) was to trace how quantum discontinuity was established in the early decades of the twentieth century. The essential datum was Planck's formula in 1900 for the distribution of energy over the different frequencies of black body radiation. In the early analysis, it was shown that assuming discontinuities in energies enabled one to deduce the Planck formula. Poincaré and Ehrenfest soon showed that the direction of deduction could be reversed.

6 I thank Pat Corvini for emphasizing this point to me.

With suitable background facts, it was possible to deduce quantum discontinuity from the evidence of the Planck formula.

4.2. Support that Is Locally Deductive but Globally Inductive

In deductive inferences, the conclusions are at best logically equivalent deductively to the premises or logically weaker than them. So it appears that deductive or near-deductive inferences to our conclusions cannot give what we seek from inductive investigations. We seek an expansion of our knowledge. These deductive inferences are merely rearranging and returning to us all or part of what we have already supposed.

This pessimistic expectation is not realized, however, once we recall that relations of support within inductive structures are not hierarchical but massively entangled. That enables the entangled relations of deductive support to combine to provide inductive support in the overall structure. This circumstance arises when we have sets of propositions that mutually support each other deductively. Nonetheless, to accept the totality is to accept propositions logically stronger than the evidence.

Striking examples of this combination of deductions arise in Newton's arguments for universal gravitation and his inverse square law of gravity. I have already sketched them above and provide a more detailed exposition in Chapter 8, "Newton on Universal Gravitation." To recall, the first example arises in his "Moon test." In it, he showed that terrestrial gravity is the same force as the celestial force holding the Moon in its orbit around the Earth. To show it, Newton reckoned that, if the force acting on the Moon strengthens with the inverse square of distance as the Earth is approached, then it would accelerate terrestrial bodies with just the accelerations actually found at the Earth's surface. The logic of the Moon test involves two hypotheses:

$H_{\text{inv. square}}$: The celestial force acting on the Moon is strengthened by an inverse square law with distance at the Earth's surface.

H_{identity}: Terrestrial gravitation and the lunar celestial force are the same.

In the context of Newton's Moon test, drawing from the evidence of the accelerations of the Moon and terrestrial bodies in free fall toward the Earth, each of these hypotheses can be deduced from the other. That is, each hypothesis provides a warrant for a deductive inference from the evidence to the other hypothesis. The two hypotheses combined are the result of the Moon test analysis. Their conjunction is inductively supported by the evidence of lunar and terrestrial accelerations.

The second example has a similar structure. The most basic results of Newton's celestial mechanics reside in two hypotheses:

$H_{ellipses}$: The planets move in their specific elliptical orbits.

$H_{inv.\ square}$: The planets are attracted to the Sun by a force that varies with the inverse square of distance.

Against the background of the observed positions of the planets and the laws of Newton's mechanics, each hypothesis could be deduced from the other. Indeed, Newton employed a subtle variant of the usual way of inferring between these two hypotheses. In the case of the near-circular orbits of the planets, he needed only the datum that the planetary orbits are reentrant. That is, in a planetary year, each planet returns to its starting point. He could then show that this reentrance was a sensitive test for deviations from the inverse square law. The observed exactness of the reentrance entailed the exactness of the inverse square law. Once again the overall inductive import of the analysis was that the evidence of the observed positions of the planets supported inductively the conjunction of the two hypotheses.

Chapter 9, "Mutually Supporting Evidence in Atomic Spectra," provides another example with a similar structure. I noted above that the Ritz combination principle enables inferences of support among the different series of the hydrogen spectrum. As detailed in the chapter, these inferences are deductive. Using the Ritz combination principle as a premise, from the Balmer series, we can deduce the Paschen, Bracket, and Pfund series. These deductions can be reversed as well. Adding the premise of only a single line from the Balmer series, we can deduce the entire, infinite Balmer series from the Paschen series. There are infinitely many series in the hydrogen spectrum, although only finitely many have been observed. The series are closely connected by further deductive relations such that we can infer deductively from any series to any other series by means of the Ritz combination principle and, if needed, the additional premise of a finite set of suitably selected lines. Although these interrelations are deductive, the final import is inductive. The Ritz combination principle and the finitely many spectral lines observed provide inductive support for the entire system of infinitely many series, each with infinitely many lines.

There might be, for some, an air of paradox in the idea that we can combine deductive relations to yield a structure with inductive import. That impression is mistaken. These cases are actually more secure inductively than many of those considered in earlier sections. In those earlier cases, inductive relations of support are combined to produce structures with overall inductive import. Inductive risk is introduced both in the component relations of inductive support and in the combined structure. If those component relations of support are deductive, then this first source of inductive risk is eliminated.

5. The Maturity of a Science

5.1. Inductive Rigidity

A preparation for the discussion of the fourth and final claim is the characterization of what constitutes mature sciences. They are characterized by inductive rigidity. That is, each proposition of the science is well supported evidentially so that a change in the proposition is not allowed by the evidence for the science. There is no assurance that a science can achieve maturity. In the early stages of the development of a science, important propositions are entertained hypothetically. They are not fixed rigidly. As the development continues, further relations of inductive support are found, the hypotheses gain evidential support, and their provisional status is discharged. If this process is completed, then the science achieves maturity such that each of its propositions is well supported.

Once this maturity is achieved, the inductive rigidity of a mature science is widely recognized among its practitioners. Challenges to the science are treated as tiresome, moribund exercises. A skeptic might doubt some proposition in a mature science. In response, someone competent in the science would be able to display the evidence that supports the proposition. In the case of special relativity, this is a dialogue with which I have some personal experience. The theory has been challenged routinely by critics since its inception over a century ago. Many of its foundational propositions have been disputed, at one time or another, unsuccessfully. The light postulate of the theory asserts that all inertially moving observers find the same speed c for light in vacuo. It is initially a puzzling postulate. Imagine an inertially moving observer chasing at high speed after a light signal that moves at c. That observer will not find the light signal slowed from c, even in the slightest.

This perplexing result makes the postulate a favored target. However, that postulate has direct support from de Sitter's analysis in 1913 of light emitted from distant double stars. Its deeper support derives from the Lorentz covariance recoverable from Maxwell's electrodynamics, in turn supported by a plethora of individual experiments in electricity and magnetism.[7]

This maturity is a goal that proponents of a theory strive to achieve, and standard textbook sciences commonly come close to achieving it. It is not uncommon, however, for the full achievement of the goal to be incomplete in parts of the theory. There propositions might achieve general acceptance while lacking proper support. The falsification of such a proposition is usually associated with great excitement and even a momentary sense of crisis. However, precisely because the falsified propositions never were strongly supported, their failure can be absorbed into theory.

On September 19, 1957, Francis Crick announced what came to be called the "central dogma" of molecular biology. It speaks, in various forms, of a unidirectional pathway of synthesis within cells from DNA to RNA to proteins. The reverse pathway is prohibited. Although the dogma was widely adopted, there was little real evidence for it. It was a simple and comfortable idea that fit with a denial of the Lamarckian inheritance of acquired characteristics.[8] When it was discovered that certain viruses could implement the reversed pathway from RNA to DNA, the result was readily incorporated into molecular biology. *Nature* published an excited editorial, "Central Dogma Reversed," in 1970.

In the twentieth century, many new particles were discovered. It was assumed routinely that the laws governing them would respect parity. That is, they would not distinguish left from right. In retrospect, there was no good evidence for this assumption other than that it had become routine in the physical laws discovered earlier. Then, in 1964, Cronin and Fitch discovered experimentally that the weak interaction in particle physics can violate charge-parity conservation. In another example, the hard-to-detect neutrinos had long been attributed to a zero rest mass. This had seemed to be a reasonable assumption. The early determinations of the neutrino rest mass pointed to a quantity in the neighborhood of zero. However, as neutrino physics developed, it became clear that a tiny mass had to be attributed to neutrinos.

7 For historical details, see Norton (2014).

8 Here I rely on Cobb (2017).

That would enable the process of neutrino oscillation in which neutrinos migrate over the three different flavors in which neutrinos manifest. This oscillation explained experimental and observational anomalies, most notably a dearth of measured electron neutrinos emitted by the Sun.[9]

In these last cases, anomalous evidence could be absorbed into the existing theories since the propositions that they contradicted lacked the strength of evidential support of other parts of the theory. Had these other better-supported parts been contradicted, the outcome would have been more troublesome. A well-supported proposition is tightly bound with so much more of the theory. Should it fail, it would bring down much more of the theory with it.

Although particle physics could absorb nonzero neutrino masses, matters would have been quite different had the OPERA Collaboration (2011) measurement been correct. Its measurements, it announced, appeared to show that neutrinos were propagating faster than light. If correct, then this would have destabilized particle physics. It would have contradicted a fundamental posit of the governing quantum field theory, the locality of quantum field operators. Particle physics was saved for the time being.

The inductive rigidity of a mature science does not make the science incorrigible. It is simply a statement of the best that can be gleaned from the evidence. No matter how strong the inductive support of a science, some inductive risk is associated with it. When incontrovertible evidence does emerge that contradicts a well-supported proposition within a mature theory, the result can be and usually is a breakdown of the theory. Rigid steel cables have some elasticity, but they will snap if overextended. What ensues is a revolution in science, a popular topic of investigation in the history of science.

These revolutions commonly occur when the science is extended beyond domains in which it was first developed and in which its evidential base is found. Newton's seventeenth-century mechanics was developed on an evidential base of slow-moving objects, such as falling stones and orbiting planets. Special relativity emerged when developments in nineteenth-century electrodynamics gave reliable results concerning much faster propagations at the speed of light. Special relativity, in turn, fails when we move to domains of intense gravitation, as Einstein found through his general theory of relativity.

9 For a review, see Gonzalez-Garcia (2003).

All of these superseded theories, however, remain evidentially well supported as long as we consider only the evidence of the domains for which they were devised. Although general relativity and relativistic cosmology now tell us that Euclidean geometry can fail when applied to spaces of cosmic extent, Pythagoras' ancient theorem remains as reliable as it ever was for the builders of houses, castles, and skyscrapers.

5.2. A Distributed Vindication

Although the inductive rigidity of a mature science is a commonplace for its practitioners, its demonstration would be a massive task. The network of interrelated propositions is enormous for any real science. A full display of the evidence and inductive relations supporting each goes well beyond what is possible in a book chapter. Indeed, for a well-developed science of great scope, displaying this rigidity in detail likely would be beyond the capacities of any single author. Rather, the requisite knowledge, though likely not fully known to any one scientist, is distributed over the full community.

This distribution is illustrated by our proper confidence in the laws of conservation of energy and momentum and our expectation that no proposal for a perpetual motion machine can succeed. Given the variety of types of proposals advanced over the centuries, a full inventory of the evidence against them would be prohibitively long. In each case, it is not enough merely to assert generically that the conservation of energy and momentum prohibits the operation of the machine. A full analysis requires us to display where the details of the mechanism proposed conflicts with other propositions in established science.[10] Different proposals will call on different expertise in the different sciences in which the proposals are formulated. We can be confident, however, that for each new proposal there is an expert in the community familiar with the pertinent science and able to respond.

A recent illustration is the "EmDrive" proposal for spaceship propulsion brought to the attention of a larger scientific community by a *New Scientist* article (Mullins 2006). It consists of microwaves in a chamber such that, it is proposed, the forces exerted by the microwaves in many directions on the chamber walls do not entirely cancel out. They leave a small net force that can propel the chamber. In this, it is unlike any other scheme of propulsion known. All known schemes produce propulsion by driving some form of

10 For a history of these proposals, see Ord-Hume (1977).

matter in the opposite direction to the thrust sought. A rocket expels hot gases. An airplane projects a current of air or hot gases behind it. A ship's propeller projects a stream of water behind it. The forward force on the rocket, airplane, or ship is balanced by an equal and opposite (reaction) force on the driven matter, as required by Newton's third law of motion. This driven matter carries rearward momentum. The conservation of momentum then assures us that the rocket, airplane, or ship gains forward momentum in the opposite direction. That is what accelerates it.

The EmDrive violates the conservation of momentum. It is a closed device supposed to set itself into motion without any ejected matter or a reactive force. Although the proposal is prima facie extremely implausible, interest in it has been remarkably stable and is matched only by the tenacity of skeptical critics. Part of the positive interest lies in wishful thinking. If it works, then it is a device that could power starships! Another reason for its endurance lies in the small magnitude of the force predicted. Detecting it requires the most delicate experiments. As critics have pointed out, such experiments can easily produce spurious results if all of the confounding effects[11] are not properly controlled.

The resulting literature is too extensive to survey here. Recounting one exchange, however, is sufficient to illustrate how the distribution of expertise works. Harold White and his collaborators at the NASA Johnson Space Center are proponents of these microwave propulsion systems. In a technical paper, White and March (2012) proposed that the reactionless thrust might arise through the Casimir force of the quantum vacuum. This is specialized physics. As they acknowledge in their introductory paragraph, classical electrodynamics precludes a reactionless force. Indeed, that classical electrodynamics conserves momentum is a result readily accessible to anyone with a serious, college-level course in electrodynamics. The Casimir effect, however, is more arcane. It is a force produced by quantum fields in a vacuum. Its basic mechanism is not so obscure. However, it is more demanding to develop a theoretical analysis of it that would securely preclude the reactionless force proposed by White and March. Such an analysis is within the expertise of Trevor Lafleur (2014), a physicist specializing in plasma physics. His analysis finds no basis for the reactionless force in the quantum vacuum.

11 Such confounders can be subtle. For example, Tajmar et al. (2018) report such a confounder in the coupling between electrical cables in the experimental setup and the Earth's magnetic field.

6. Inductively Self-Supporting Structures

There are self-supporting inductive structures.

6.1. Inductive Closure: That Is All There Is

A self-supporting inductive structure is a set of propositions such that each one in the set is well supported evidentially; the evidence supporting it is in the set of propositions; and the propositions that warrant the relations of inductive support are propositions within the set. This set is inductively closed.

We have already seen such self-supporting inductive systems in the small. If we take the background propositions from among which they proceed as fixed, then they are found in the examples above of pairs of mutually supporting hypotheses and of networks of inductive support such that the relations of support cross over one another in a bewildering tangle. The more difficult and interesting problem is whether such systems arise on the large scale and whether they are embodied by our mature sciences. I will argue in the subsection below that, if a mature science is properly characterized by the rigidity described in the previous section, then the material theory entails that it is a self-supporting inductive structure.

Before proceeding, it will be helpful to address directly the sense that such structures are paradoxical. They might sound akin to lifting oneself into the air by pulling on one's own bootstraps. However, there is no paradox. If one can affirm that each proposition in the set is well supported individually in virtue of other propositions in the set, then there is nothing more that can be asked. The analogy to pulling oneself up by one's own bootstraps fails.[12] A better architectural analogy is to some elaborate sculpture whose total stability appears to be impossible, yet it still stands. A simple example is the tensegrity icosahedron of Figure 2.5.

12 In the imagined scenario, we hover in midair by pulling on our bootstraps. The pulling force is supposed to counter the force of gravity. This analysis neglects another force. The upward force from the bootstraps in tension is balanced by the downward force from the corresponding compression in our legs. The force of gravity remains unbalanced, and the eager bootstrap puller falls to the Earth.

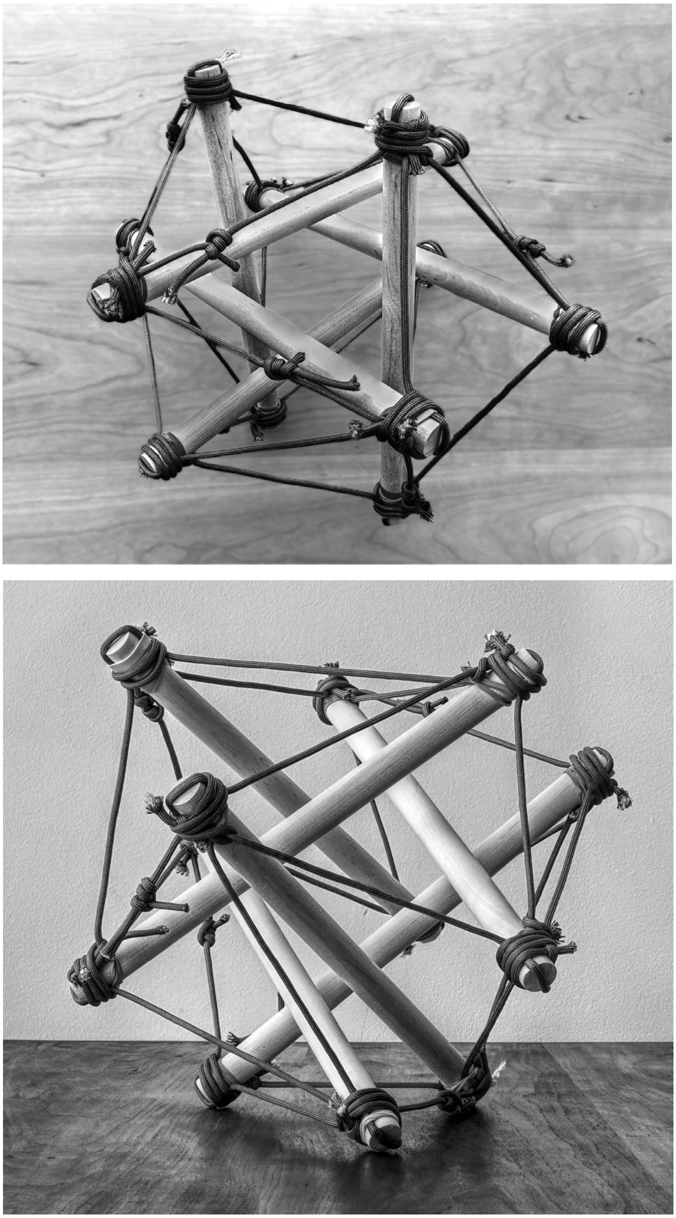

Figure 2.5. Plan and elevation of a tensegrity icosahedron; model and photographs by John D. Norton

In a superficial description, it seems to be impossible that such a tensegrity structure can stand. There are six rods connected only by cords in tension. One end of each of three rods rests on the table surface. All of the remaining rods and their parts are held suspended above the table surface. No rod directly touches any other rod. Their *sole* connections are through cords in tension. Such a structure, it seems, would collapse into a pile of rods and cords. Must not a rod, supported only by cords in tension, anchor those cords on another rod still higher in the structure? And must not that rod be held by cords tied to another still higher rod? And so on in an infinite regress? Yet there are only six rods, and it stands.

On closer examination, we can inspect any rod individually and affirm that it is supported securely by cords attached to both ends. That is true for any rod that we examine. That is all that is needed for the structure to stand. We need no additional, holistic condition beyond the condition that each rod individually is supported.

It is the same with self-supporting inductive structures. If we can affirm that each proposition individually is well supported inductively, then nothing further need be demanded. Of course, if we were tacitly to assume a hierarchical structure for relations of inductive support, then these self-supporting inductive structures are impossible. Then at least some of the propositions needed to warrant all of the inductive inferences in the structure could not be inductively supported themselves within a finite structure. An infinite regress would ensue. However, as argued in detail above, this hierarchical assumption is incorrect.

One might still harbor reservations. These self-supporting inductive structures necessarily harbor circularities in the relations of support. That these circularities are benign I argue at length in Chapter 3, "Circularity." Or one might accept that such structures exist but that they make the import of evidence equivocal since our evidence might support many such systems. In Chapter 4, "The Uniqueness of Domain-Specific Inductive Logics," I argue that a mechanism, native to the material theory of induction, precludes this danger.

6.2. Mature Sciences Are Self-Supporting Inductive Structures

We can now see that mature sciences are inductively self-supporting. This conclusion requires that the compass of a mature science is expanded enough, possibly even to embrace neighboring sciences, so that inductive closure is secured. That means that we can select any proposition in the mature science and will find, within that compass, the evidence that inductively supports the proposition along with the propositions that warrant the inductive support.

This assertion of self-support supposes that we can expand the compass of a mature science sufficiently to secure closure. We can imagine that sequences of inductive inferences and warranting propositions form an infinite chain that outstrips finite description so that no finite expansion is adequate. I do not see how, as a matter of inductive logic, such a chain can be dismissed without further examination of its details. Perhaps it is possible. However, I do not see that it arises in actual practice in our mature sciences. If that were the case, then the inductive rigidity of a mature science would not be accessible to us. Yet our repeated experience in the history of science is that we do have mature sciences that display just the inductive rigidity described here.

7. Nonempirical Components of the Large-Scale Structure of Inductive Support

This chapter provides an account of the large-scale structure of inductive support that uses only materially warranted inductive inferences or relations of inductive support. One might accept that much of this large-scale structure is captured by the material theory. However, it might be tempting to imagine that the material account still needs to be supplemented by deeper, nonempirical truths if the account of the large-scale structure is to be complete. Such deeper truths would be beyond normal evidential scrutiny and thus outside the reach of the material theory.

To make it plausible that no such added components are viable, in this section I consider and reject some candidates.

7.1. The Universal Logic of Induction

The least adventurous proposal for the added component is that the large-scale structure requires at least some universal rules of inductive inference or some general calculus of induction. Perhaps we do need to assume the universal applicability of the probability calculus to all relations of inductive support, as Bayesians seem to hold. The failure of all such universal rules has been argued at length in the *Material Theory of Induction* (Norton 2021) and reviewed in Chapter 1. There is no need for these arguments to be repeated here.

7.2. Kantian Synthetic, A Priori Propositions

Might the very viability of induction at all depend on a Kantian synthetic a priori proposition? Such a proposition would be factual, but it would require no evidence since its truth — supposedly — can be established a priori: that is, independently of experience. Since the literature on this one idea could occupy many lifetimes, I dare express only my view that this literature has failed to provide viable examples of synthetic a priori propositions that could serve this function. Kant's original proposals did not fare well. It might have been appealing to imagine in the eighteenth century that, as an a priori certainty, space could never manifest to us other than as Euclidean. However, those who have absorbed the variant spatial geometries brought by general relativity find it otherwise. The geometry of space is not something determinable a priori but a subject for empirical investigation.

The mode of failure of this one proposal for a synthetic a priori proposition afflicts all of the proposals. If they make a definite, factual assertion, then they end up failing empirically. If they escape empirical refutation by vagueness, then they make no factual assertion and are empty.

7.3. Causality

Might we seek such a condition in a principle of causality? It is a Kantian principle and has an enduring popularity outside Kantian circles. The principle asserts that every effect is brought about in a regular manner by some cause. Might such a supposition be a precondition for science and thus for inductive inferences in science? I have criticized this conception at length elsewhere.[13] In short, the problem is that the terms "cause" and "effect" are so poorly specified that the principle is factually vacuous. We can always implement the principle in any scenario simply by artful choices for what the terms designate. Things in the world do connect in a myriad of interesting ways. What those ways are cannot be stipulated a priori but must be discovered empirically.

7.4. Mathematics

It is often found remarkable that mathematical descriptions of the world are so fertile and powerful. Might the supposition of a mathematical structure

13 See, for example, Norton (2003, 2016, 2024).

of the world be a prior condition necessary at least for the physical sciences? There is much to say on this supposition. The main point of relevance is that the supposition itself is open to empirical testing. We have tested it and found that it applies to a surprisingly large range of phenomena. This means that, in the absence of any deeper, a priori vindication, it is a contingent fact to be learned inductively. In this regard, it is no different from the other warranting facts of the physical sciences. It is not an obstacle to the material warranting of inferences but a part of it.

An illustration shows how the proposition is not an a priori principle but open to the possibility of failure empirically. Much of modern physics presumes that its basic laws are to be written as differential equations. That fundamental presumption has been challenged by Stephen Wolfram (2002). His "new kind of science" seeks to replace these differential equations in physics by discrete algorithms and cellular automata. It is a most radical proposal. Wolfram has continued to press his approach, but its reception among physicists remains poor. Their skepticism is not based on an assertion that, as an a priori matter, the physical world must be governed by differential equations. Rather, as Becker (2020) reports briefly, the doubt is that Wolfram's methods can recover the present results of physics with the same scope and accuracy. The concern is empirical. The proposal lacks powerful enough inductive support to supplant existing methods.

Nonetheless, we can still ask what the prospects are for an a priori justification for the mathematical character of nature. These prospects are poor, in my view, since it is doubtful that there is a deep truth in the supposed mathematical character of nature. Rather, I harbor an enduring concern that our deference to the power of mathematical descriptions is excessive. The supposed truth is empty unless the specific mathematics favored by nature is specified. Yet the only way that we know to identify the right mathematics among many choices is empirical. Thus, I find it hard to be moved by a celebrated and poetic confession attributed to Heinrich Hertz: "One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them."[14] On the contrary, I am in awe not of the formulae but of the creativity

14 As quoted in Bell ([1937] 1953, 16). The quotation is unsourced and seems to be the origin of later repetitions. Recently, Shour (2021) has tracked down the origin of the remark in

of mathematicians who made them. New physical theories commonly come in clumsy mathematical clothing. Each new physical theory is taken as a challenge by mathematicians to find formulations in which the new theory looks mathematically simple and natural. The ensuing mathematics fits the world not through some preordained harmony but merely retrospectively through our ingenious and artful contrivances.[15]

To see the process, one need only recall the inadequacies of geometry as Euclid formulated it for the celestial mechanics of the seventeenth century. Kepler sought to use the Platonic solids in a nestled geometric structure to explain the relative orbital sizes of the planets. We now regard the whole project not as reflecting the inner mathematical constitution of the world but as dependent on barren mathematical coincidences. One can only wonder at Newton's labors in his *Principia* to develop his celestial mechanics using simple Euclidean geometry so poorly suited to the task. The theory becomes so much more elegant and transparent when re-expressed in the later methods of vector calculus, contrived in part precisely for this purpose.

7.5. The Ineffable

Finally, when explicit attempts to identify these nonempirical conditions fail, one might be tempted by the idea that these conditions are present but ineffable. They are so deeply enmeshed in our ways of thinking that, it is speculated, we cannot discern them. This appears to me to be the last defense of a failing program. These conditions have powerful consequences in connecting facts, and these connections are fully accessible to us. Yet the conditions that underwrite these connections are supposed to be opaque to us. The supposition of their invisibility makes them irrelevant. What matters are the contingent connections that they supposedly induce among the facts of the science, and we can be secure in accepting these connections only if we can affirm or support them through methods accessible to us.

Hertz's published writings. (I thank Marc Lange for letting me know of Shour's paper.)

15 For another expression of this view in counterpoint to Einstein's later Platonism, see Norton (2000, Appendix D).

8. Conclusion

The four claims defended in this chapter form the basis of the material understanding of the large-scale structure of relations of inductive support. These claims by no means exhaust the questions that one might raise about this large-scale structure and the accompanying skeptical challenges to the material understanding. Some of these questions and challenges will be raised in the chapters to come in Part I, and the claims defended in this chapter will be used to answer them. I will ask in Chapter 3, if the structure is nonhierarchical, does it harbor circularities? (Yes.) Are they benign? (Yes.) What of uniqueness? I will ask in Chapter 4. That is, can a finite body of empirical evidence, even if extensive, yield a unique, self-supporting structure? (Yes.) Or must we forever contend with multiple, competing, self-supporting structures? (No.) Relations of inductive support are nonhierarchical and circular. Does this mean, I will ask in Chapter 5, that the material theory of induction is just a coherentist epistemology? (No.) And finally, in Chapter 6, what of *the* problem of induction? Is the material theory prone to the traditional problem? (No.) Is there an analogous problem residing in a fatal regress of warrants? (No.)

These are all good questions and worthy challenges. I will show that the material approach to inductive inference has ample resources for answering them.

REFERENCES

Anonymous. 1970. "Central Dogma Reversed" (editorial). *Nature* 226: 1198–99.

Becker, Adam. 2020. "Physicists Criticize Stephen Wolfram's Theory of Everything." *Scientific American*, May 6, 2020. https://www.scientificamerican.com/article/physicists-criticize-stephen-wolframs-theory-of-everything/.

Bell, Eric Temple. (1937) 1953. *Men of Mathematics*. Vol. 1. London: Penguin.

Bondi, Hermann, and Thomas Gold. 1948. "The Steady-State Theory of the Expanding Universe." *Monthly Notices of the Royal Astronomical Society* 108: 252–70.

Boyd, Nora. 2018a. "Scientific Progress at the Boundaries of Experience." PhD diss., University of Pittsburgh.

———. 2018b. "Evidence Enriched." *Philosophy of Science* 85: 403–21.

Cobb, Matthew. 2017. "60 Years Ago, Francis Crick Changed the Logic of Biology." *PLoS Biology* 15, no. 9: e2003243.

Dalton, John. 1808. *New System of Chemical Philosophy. Part I.* Manchester: R. Bickerstaff.

Gonzalez-Garcia, M.C. 2003. "Neutrino Masses and Mixing: Evidence and Implications." *Reviews of Modern Physics* 75: 345–402.

Lafleur, Trevor. 2014. "Can the Quantum Vacuum Be Used as a Reaction Medium to Generate Thrust?" https://arxiv.org/pdf/1411.5359.pdf.

Mullins, Justin. 2006. "Relativity Drive: The End of Wings and Wheels." *New Scientist* 2568. https://www.newscientist.com/article/mg19125681-400-relativity-drive-the-end-of-wings-and-wheels/.

Norton, John D. 1993. "The Determination of Theory by Evidence: The Case for Quantum Discontinuity 1900–1915." *Synthese* 97: 1–31.

———. 2000. "'Nature in the Realization of the Simplest Conceivable Mathematical Ideas': Einstein and the Canon of Mathematical Simplicity." *Studies in the History and Philosophy of Modern Physics* 31: 135–70.

———. 2003. "Causation as Folk Science." In *Causation, Physics and the Constitution of Reality*, edited by Huw Price and Richard Corry, 11–44. Oxford: Oxford University Press.

———. 2005. "A Little Survey of Induction." In *Scientific Evidence: Philosophical Theories and Applications*, edited by Peter Achinstein, 9–34. Baltimore: Johns Hopkins University Press.

———. 2014. "Einstein's Special Theory of Relativity and the Problems in the Electrodynamics of Moving Bodies that Led Him to It." In *Cambridge Companion to Einstein*, edited by Michel Janssen and Christoph Lehner, 72–102. Cambridge, UK: Cambridge University Press.

———. 2016. "Curie's Truism." *Philosophy of Science* 83: 1014–26.

———. 2021. *The Material Theory of Induction*. BSPS Open Series. Calgary: University of Calgary Press.

———. 2023. "Inductive Inferences on Galactic Redshift, Understood Materially." In *Current Debates in Philosophy of Science*, edited by C. Soto, Synthese Library 477, 227–46. Cham, Switzerland: Springer.

———. 2024. "The Metaphysics of Causation: An Empiricist Critique." In *Alternative Approaches to Causation*, edited by Yafeng Shan, Chapter 4. Oxford: Oxford University Press.

OPERA Collaboration. 2011. "Measurement of the Neutrino Velocity with the OPERA Detector in the CNGS Beam." https://arxiv.org/abs/1109.4897.

Ord-Hume, Arthur W.J.G. 1977. *Perpetual Motion: The History of an Obsession*. London: George Allen and Unwin.

Shour, Robert. 2021. "Heinrich Hertz on the Wisdom of Equations." Preprint at https://www.researchgate.net/publication/351564092_Heinrich_Hertz_on_the_wisdom_of_equations.

Smith, George E. 2002. “The Methodology of the Principia.” In *The Cambridge Companion to Newton*, edited by I. Bernard Cohen and George E. Smith, 138–73. Cambridge, UK: Cambridge University Press.

———. 2014. “Closing the Loop: Testing Newtonian Gravity, Then and Now.” In *Newton and Empiricism*, edited by Zvi Biener and Eric Schliesser, 262–350. Oxford: Oxford University Press.

Tajmar, Martin, Matthias Kößling, Marcel Weikert, and Maxime Monette. 2018. “The SpaceDrive Project — First Results on EMDrive and Mach-Effect Thrusters.” https://tu-dresden.de/ing/maschineanwesen/ilr/rfs/ressourcen/dateien/forschung/folder-2007-08-21-5231434330/ag_raumfahrtantriebe/SPC-The-SpaceDrive-Project-First-Results-on-EMDrive-and-Mach-Effect-Thrusters.pdf?lang=en.

White, Harold, and Paul March. 2012. “Advanced Propulsion Physics: Harnessing the Quantum Vacuum.” https://www.lpi.usra.edu/meetings/nets2012/pdf/3082.pdf.

Wolfram, Stephen. 2002. *A New Kind of Science*. Champaign, IL: Wolfram Media.